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Subject: Fluid Mechanics

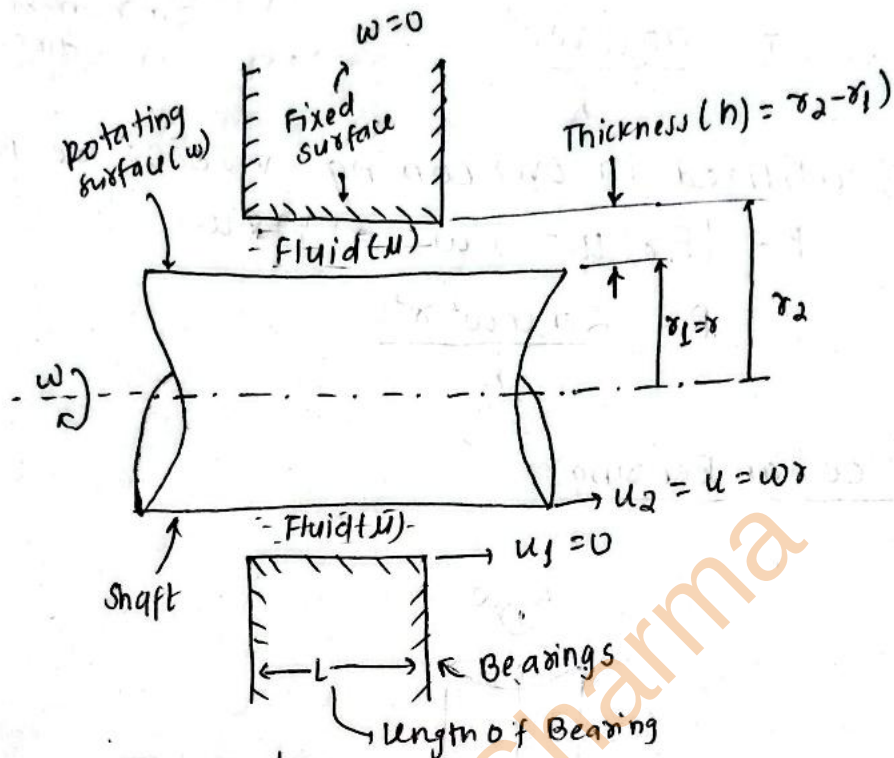
(Safal chettri (PBE) sir)

Surendra Sharma



Bearings

(I) Journal (or Bush) Bearings-



$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{(u_2 - u_1)}{(r_2 - r_1)} \Rightarrow \tau = \mu \frac{u}{h} = \mu \left(\frac{\omega r}{h} \right)$$

Since,

Tangential (or Peripheral velocities):

$$u_1 = 0 \text{ and } u_2 = \omega r = u$$

Since,

$$\omega = \frac{2\pi N}{60}$$

$N \rightarrow$ No. of revolution per minute (rpm)

✓ Frictional (or shear) force, $F = \tau \frac{A_c}{L}$ Contact Area

$$\therefore F = \frac{2\pi\mu\omega r^2 L}{h}$$

Perimeter (P) $\times$ Length (L)

$$2\pi r \times L$$

✓ Torque (T) required to maintain an angular velocity (ω) of rotating cylinder

$$T = Fx$$

$$\therefore T = \frac{2\pi\mu\omega R^3 L}{h}$$

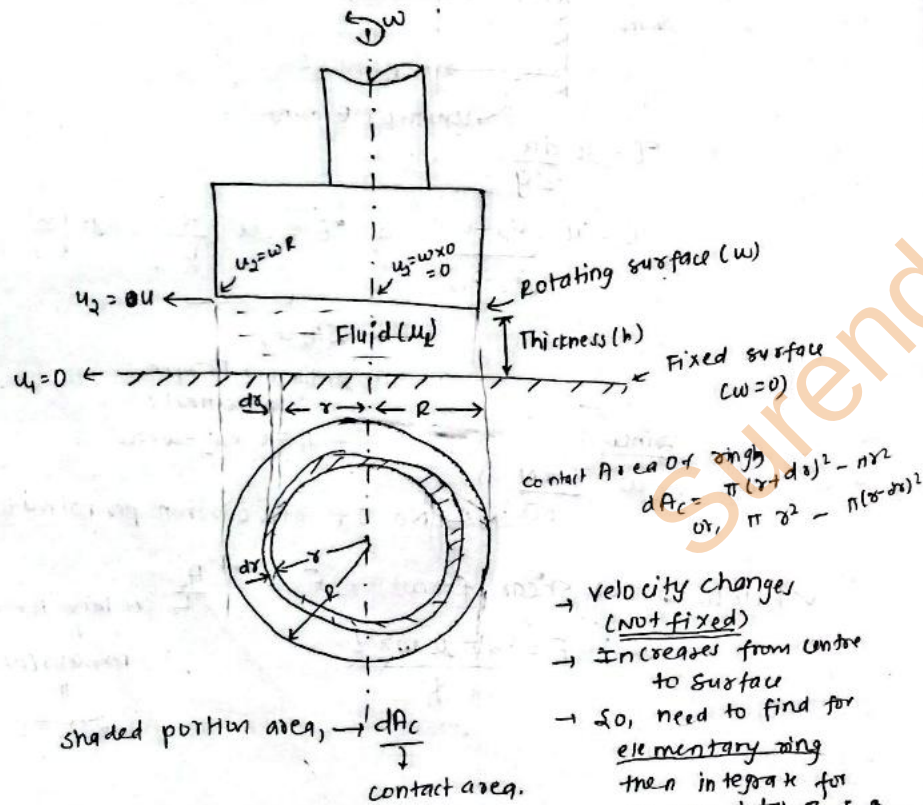
{ In this case velocity u_1 is constant, so, can be used directly }

✓ Power (P) utilized in overcoming viscous resistance:

$$P = (Fx)\omega = T\omega$$

$$\therefore P = \frac{2\pi\mu\omega^2 R^3 L}{h}$$

(II) Foot (or collar) Bearing



→ velocity changes (not fixed)
→ Increases from centre to surface
→ So, need to find for elementary ring then integrate for finding total T, F, P

$$r: 0 \rightarrow R$$

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{(u_2 - u_1)}{y_2 - y_1}$$

$$= \mu \frac{u}{h}$$

$$\tau = \mu \left(\frac{\omega r}{h} \right) \quad \text{--- (1)}$$

✓ Frictional (or shear force), F , on elementary ring of width dr :

$$dF = \tau dA_c$$

$$dF = \mu \left(\frac{\omega r}{h} \right) \cdot 2\pi r dr$$

{ $\therefore$ contact area of elementary ring, $dA_c = 2\pi r dr$ }

$$\therefore dF = \frac{2\pi\mu\omega r^2 dr}{h}$$

✓ Torque (T) req. to maintain an angular velocity (ω) of rotating cylinder.

$$dT = dF r$$

$$\therefore dT = \frac{2\pi\mu\omega r^3}{h}$$

$$\therefore T = \int_0^R dT = \int_0^R \frac{2\pi\mu\omega r^3}{h} dr = \frac{2\pi\mu\omega R^4}{4h}$$

$$\therefore T = \frac{\pi\mu\omega R^4}{2h}$$

First check whether velocity is constant or not
If constant $\rightarrow$ done
Not constant $\rightarrow$ elementary $\downarrow$ Integrate

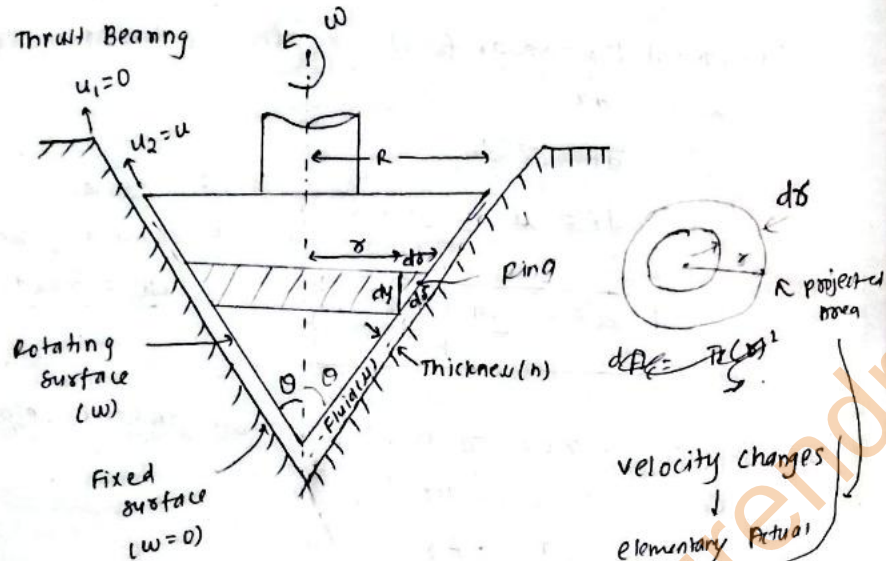
✓ Power (P) utilized in overcoming the viscous resistance in rotating on foot bearing.

$$P = (Fr)u = Tw$$

$$P = \frac{\pi \mu \omega^2 R^4}{2h}$$

{thickness
↳ to fixed surface}

(III) Conical Thrust Bearing



Newton's law of viscosity,

$$\begin{aligned} \tau &= \mu \frac{du}{dy} \\ &= \mu \frac{(u_2 - u_1)}{h} \\ &= \mu \frac{u}{h} \\ \tau &= \mu \left(\frac{\omega r}{h} \right) \quad \text{--- (1)} \end{aligned}$$

velocity changes
↓
elementary Area
(really fluid at touch with parts)

Surface Area of ring (fluid at touch),
{Contact area, $dA_c = 2\pi r \cdot ds$
Actual, fluid is touch ज़रूरी surface πds से
So, $dA_c = 2\pi r \cdot \frac{ds}{\sin \theta}$

ds को dr में convert करें
↳ as $r: 0 \rightarrow R$ $ds \rightarrow ??$
↳ change

$$\sin \theta = \frac{dr}{ds}$$

$$ds = \frac{dr}{\sin \theta}$$

Thus,
 $dA_c = 2\pi r \cdot \frac{dr}{\sin \theta}$

✓ Frictional force (or shear) on elementary ring of width dr :

$$dF = \tau \cdot dA_c = \frac{2\pi \mu \omega r^2}{h} \cdot ds$$

$$dF = \frac{2\pi \mu \omega r^2}{h} \left(\frac{dr}{\sin \theta} \right) \quad \left\{ \begin{array}{l} \text{Contact area of} \\ \text{elementary ring} \\ dA_c = 2\pi r \cdot ds \end{array} \right.$$

✓ Torque (T) required to maintain an angular velocity (ω) of rotating cylinder:

$$dT = dFr$$

$$dT = \frac{2\pi \mu \omega r^3}{h \sin \theta} dr$$

$$T = \int_0^R dT = \int_0^R \frac{2\pi \mu \omega r^3}{h \sin \theta} dr$$

$$T = \frac{2\pi \mu \omega R^4}{4h \sin \theta}$$

$$T = \frac{\pi \mu \omega R^4}{2h \sin \theta}$$

✓ Power (P) utilized in overcoming the viscous

$$P = (F \cdot u) = T \omega$$

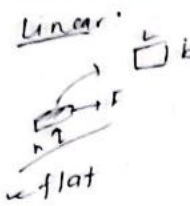
$$\therefore P = \frac{\pi \mu \omega^2 R^4}{2h \sin \theta}$$

{ Inclined contact Area }

Note:

For the calculation of problems related to viscosity following steps should follow,

- First of all, it is necessary to identify contact area with liquid (A_c)
- Area should be linear (i.e. flat surface), circumferential (i.e. cylindrical or conical).
- Identify the velocity in the contact Area, whether it is linear or angular.
- If it is angular, find linear velocity.
- See whether the linear velocity is constant or not throughout the contact area.
- If it is constant throughout the contact area there, we can use it directly in equation and find required parameters.
- If it is varying throughout the contact area, it is necessary to integrate the velocity throughout the contact area.
- Finally, we can use this velocity and find required parameters.



$$\text{sp. wt. } \gamma = \frac{\rho \times g}{g = 9.81 \text{ m/s}^2}$$

$$\text{sp. gravity} = \frac{\gamma_L}{\gamma_w} = \frac{\gamma_L}{\gamma_w}$$

Q. At a depth of 9km in the ocean, the pressure is 82 MPa. Assume sp. wt. at surface as 10 kN/m³ and that the avg. bulk modulus of elasticity is 2.4 x 10⁹ N/m². Calculate

- change in sp. vol. b/w that at surface & at that depth,
- sp. volume at that depth,
- sp. wt. at that depth.

→ Soln.

$$h = 9 \text{ km} = 9000 \text{ m}$$

$$P_{\text{km}} = 82 \text{ MPa}$$

$$P_{\text{surface}} = 0 \text{ Pa (Gauge pressure w.r.t. Atm. pressure)}$$

$$\gamma_{\text{surface}} = 10 \text{ kN/m}^3$$

$$K = 2.4 \times 10^9 \text{ N/m}^2$$

$$K = \frac{dP}{-\left(\frac{dV}{V_1}\right)} \quad (i) \quad \text{Thus, } 2.4 \times 10^9 = \frac{82 \times 10^6}{-\left(\frac{dV}{V_1}\right)}$$

$$\frac{\text{sp. volume}}{\frac{1}{\gamma}}$$

$$\gamma = \frac{\rho \times g}{g}$$

$$V_1 = \frac{1}{\gamma} = \frac{g}{\gamma_1} = \frac{9.81}{10 \times 10^3} = 9.81 \times 10^{-4}$$

$$\frac{dV}{\text{change in sp. vol.}} = -3.35 \times 10^{-5} \text{ m}^3/\text{kg}$$

$$(ii) \quad \gamma_{gem} = \gamma_2 = \frac{g}{V_2}$$

$$V_2 = \frac{1}{S_2} = \frac{g}{\gamma_2}$$

$$dV = V_2 - V_1^{surface}$$

$$-3.35 \times 10^{-5} = V_2 - 3.981 \times 10^{-4}$$

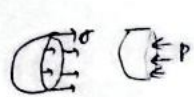
$$Sp. volume \leftarrow V_2 = 9.475 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$(iii) \quad \frac{\gamma_{gem}}{1} = \gamma_2 = \frac{g}{V_2} = \frac{9.81}{9.475 \times 10^{-4}} = \frac{10353.56}{1} = 1036 \text{ kN/m}^2$$

Sp. wt

Q. What should be diameter of a droplet of water, if the pressure inside is to be $0.0018 \text{ kg(f)/cm}^2$ greater than outside given value of surface tension of water in contact with air at 20°C as 0.0075 kg(f)/m (1.67mm)

→ Solⁿ: For droplet of water,



$$P_{diff} = \frac{4\sigma}{d}$$

$$or, \frac{0.0018}{10^{-4}} = \frac{4 \times 0.0075}{d}$$

$$\Rightarrow d = 1.67 \times 10^{-3} \text{ m}$$

$$d = 1.67 \text{ mm}$$

$$[1N = \frac{1 \text{ kg(f)} \times 9.81}{1000}]$$

$$\frac{\text{kg(f)}}{1000}$$

Force given in kg

(Tyo khatatki man)

So,
In N,

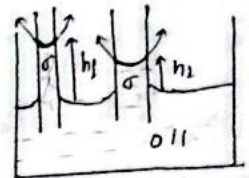
$$1N = 1 \text{ kg(f)} \times 9.81$$

$$Press \times Area = \sigma \times \text{Perimeter}$$

5. Two capillary tubes of diameter 1mm and 2mm are placed in an oil of surface tension 0.04 N/m and sp. wt of 9400 N/m^3 . Find diff. of oil levels in two tubes. Assume, contact angle = 25° .

→ Solⁿ:

$$h = \frac{2\sigma \cos \theta}{\rho r g}$$



$$\sigma \times 2 \cos \theta$$

$$\gamma_{oil} \times Vol = \sigma \cos \theta \times 2\pi r$$

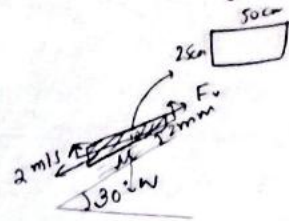
$$9400 \times (\pi r^2) \times H_1 = 2\pi r \sigma \cos \theta$$

$$H_1 = \frac{2\sigma \cos \theta}{\gamma_{oil} r_1} = 7.71 \times 10^{-3} \text{ m}$$

$$H_2 = \frac{2\sigma \cos \theta}{\gamma_{oil} r_2} = 3.85 \times 10^{-3} \text{ m}$$

$$\text{Diff of oil levels} = H_1 - H_2 = 3.85 \times 10^{-3} \text{ m}$$

Q A rectangular plate of size 25cm by 50cm and weighing 250N slides down a 30° inclined surface at a uniform velocity of 2m/s. If uniform 2mm gap b/w the plate and inclined surface is filled with oil, determine viscosity of oil.



$$W \sin 30^\circ = F_v$$

$$\text{or, } 250 \sin 30^\circ = \mu \times \frac{du}{dy} \times A_{\text{contact}}$$

$$\text{on } 250 \sin 30^\circ = \mu \times \frac{(2-0)}{2 \times 10^{-3}} \times (0.25 \times 0.50)$$

$$\mu = 3 \text{ Pa.s} = 1 \text{ N/m}^2$$

Q 0.02 litres of a liquid ($\rho = 0.9$) flow in 40 sec through a pipe 0.25m long and 15mm diameter emerging from the bottom of a tank holding the liquid maintained at a constant level of 0.35m above the outlet end of pipe discharging freely. Compute viscosity of liquid.

→ Soln:

$$H_L = \text{Head loss} = \frac{32 \mu \cdot V \cdot L}{\gamma D^2}$$

$$0.35 \text{ m}$$

$$\text{Velocity, } V = \frac{\frac{\text{Volume}}{\text{time}}}{\frac{\pi d^2}{4}} = \frac{0.02 \times 10^{-3}}{40 \times \frac{\pi \times (15 \times 10^{-3})^2}{4}} = 0.2829 \text{ m/s}$$

$$S = \frac{\gamma_L}{\gamma_w} \rightarrow 0.9$$

$$\mu = 3.072 \times 10^{-3} \text{ N/m}^2$$

$$= 3.072 \times 10^{-2} \text{ poise}$$

Q A sphere of diameter 2mm falls 150mm in 50 seconds in a viscous liquid. The density of sphere is 7500 kg/m³ and liquid is 900 kg/m³. Find coefficient of viscosity of liquid (19.17 poise)

$$\text{Soln: } D = 2 \text{ mm} \quad t = 50 \text{ sec}$$

$$L = 150 \text{ mm}$$

$$F_{\text{upthrust}} + \text{viscous force} = \text{Weight}$$

$$\gamma_L$$

$$\text{Stoke's law}$$

$$\gamma_L \times V_s + 3\pi \mu D V = \gamma_s \times V_s$$

$$\gamma_L \times V_s$$

$$V_s = \frac{1}{6} \pi D^3 = \frac{1}{6} \pi \times (2 \times 10^{-3})^3 = 4.18 \times 10^{-9} \text{ m}^3$$

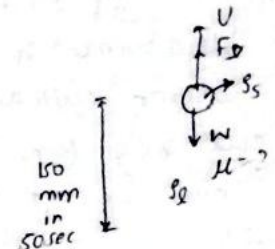
$$V = \frac{150 \times 10^{-3}}{50} = 3 \times 10^{-3} \text{ m/s}$$

$$900 \times 9.81 \times 4.18 \times 10^{-9} + 3\pi \mu \times 2 \times 10^{-3} \times 3 \times 10^{-3} = 7500 \times 9.81 \times 4.18 \times 10^{-9}$$

$$\mu = \frac{2.706 \times 10^{-4}}{3\pi \times 2 \times 10^{-3} \times 3 \times 10^{-3}}$$

$$= 1.914 \text{ N/m}^2$$

$$= 19.14 \text{ poise}$$



(Falling sphere viscometer)

(Rotating cylinder viscometer)

Q9. The viscosity of a liquid is determined by rotating cylinder method, in which case the inner cylinder of diameter 20cm is stationary. The outer cylinder diameter 20.5cm, contains the liquid upto ht 30cm. The clearance at bottom of two cylinders is 0.5cm. The outer cylinder is rotated at 400 rpm. The torque registered on the torsion meter attached to inner cylinder is 5.886 Nm. Find viscosity of liquid.

→ Sol:

$$\tau = \mu \frac{du}{dy}$$

$$= \mu (u_2 - u_1) \frac{1}{dy}$$

$$= \mu \times \frac{\omega R_2}{2h}$$

$$= \mu \times \frac{400 \times 20.5}{2 \times 0.5}$$

Contact Area (A_c)

$$= 2\pi R_2 \times L_c$$

$$= 2\pi \times 10.25 \times (30 - 0.5)$$

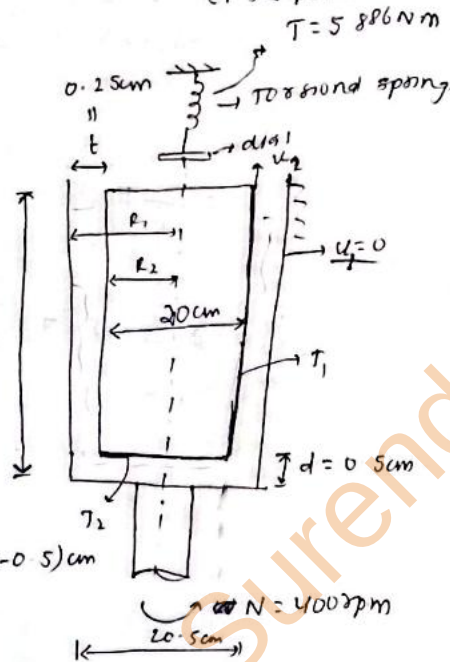
$$= 0.185 \text{ m}^2$$

$$F = \tau A_c = 317.66 \text{ mN}$$

$$T_1 = F \times R_{in} = 31.76 \text{ Nm}$$

$$T_2 = \frac{\pi \mu \omega R_{in}^4}{2h}$$

$$T = T_1 + T_2 = 5.886$$



$$T = T_1 + T_2 = 5.886$$

$$\text{or, } \frac{F \times R_{in}}{2A_c} + \frac{\pi \mu \omega R_{in}^4}{2h} = 5.886$$

$$\text{or, } 309.97 \times 0.1 \mu + 0.044 \mu = 5.886$$

$$\mu = 1.896 \text{ Poise}$$

Q Find the power dissipated in the foot step bearing where radius of foot step disc is 10cm. An oil of viscosity 80 centipoises is filled in b/w the rotating disc and fixed bearing surface. The thickness of oil film and normal speed of disc is 1000 rpm and 2mm respectively.

→ Sol:

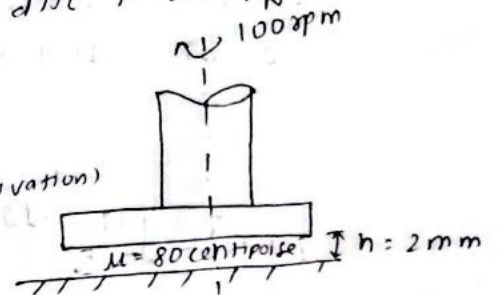
$$T = \frac{\pi \mu \omega R_{in}^4}{2h}$$

$$= 6.578 \text{ Nm} \times 0.0658$$

$$P = T \omega$$

$$= 0.688 \text{ W}$$

(Along with quick formula derivation)



$$\mu = 80 \text{ cP} = 0.08 \text{ Pa.s}$$

$$\omega = \frac{2\pi N}{60} = 104.72 \text{ rad/s}$$

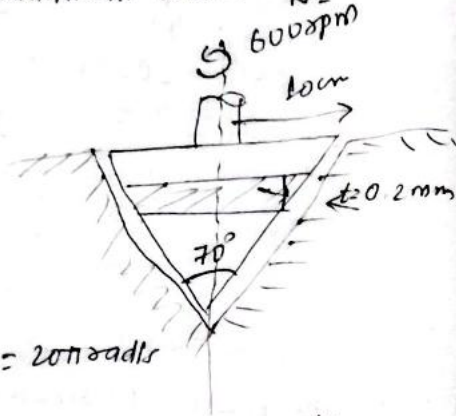
$$R = 10 \text{ cm} = 0.1 \text{ m}$$

$$h = 2 \text{ mm} = 0.002 \text{ m}$$

$$T = \frac{\pi \mu \omega R^4}{2h} = 6.578 \text{ Nm}$$

$$P = T \omega = 0.688 \text{ W}$$

Q12 Find the viscosity of an oil which is filled in b/w the rotating solid cone and fixed bearing surface.
 Given, power dissipated = 100W, Normal speed = 600 rpm.
 thickness of oil film = 0.2 mm, maximum radius of solid cone = 10 cm
 and apex angle of cone = 70°
 → Soln



$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{\omega r}{t}$$

$$\omega = \frac{2\pi N}{60} = 20\pi \text{ rad/s}$$

$$\tau = 100000\pi\mu r$$

$$dT = dF r$$

$$dT = \tau dA_c r$$

$$\text{or } dT = 100000\pi\mu r^2 \times 2\pi r dr$$

$$dT = ds \sin \alpha$$

$$T = \int dT = \frac{1973920.88 \mu}{\sin(35^\circ)} \int_0^{0.1} r^3 dr$$

$$T = 86.0356 \mu$$

$$P = T\omega = 86.0356 \mu \times 20\pi$$

$$\frac{100}{100} = 5405.78 \mu$$

$$\therefore \mu = 0.0185 \text{ N s/m}^2$$

$$\mu = 0.185 \text{ Poise.}$$

③

- Fluid Pressure at point

③ # Fluid Pressure and

• Fluid Pressure 'Pressure', or 'intensity of pressure'

↓

$$\frac{\text{Force}}{\text{Area}}$$

$\downarrow$
 $\frac{\text{Force}}{\text{Area}}$ (Force exerted on unit area)
is uniformly distributed

✓ However if the forces is non uniformly distributed, the expression will give the average value only.

✓ When pressure varies from point to point on an area, $p = dF / dA$ [∵ 'dF' represents

Pressure at any point, $P = \frac{dF}{dA}$ [$\because$ 'dF' represents force acting on an infinitesimal area 'dA']

SI unit $\rightarrow \text{N/m}^2$ (or Pascal)

✓ pressure force act normally

- Pascal's law

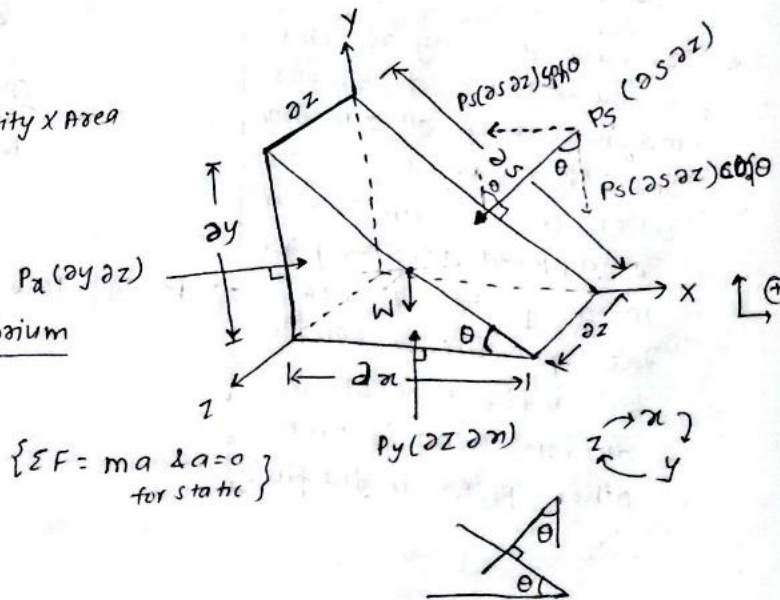
Force = Pressure intensity $\times$ Area

✓ In static Equilibrium

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\left\{ \sum F = ma \text{ \& } a=0 \right\}$$



$\sum F_x = 0$
 $-F_x (2x \cdot 2z) \sin \theta + F_y (2y \cdot 2z) \sin \theta = 0$
 $\Rightarrow -F_x (2x \cdot 2z) \frac{2y}{2} + F_y (2y \cdot 2z) \frac{2x}{2} = 0$

$\Rightarrow (F_x - F_y) 2y \cdot 2z = 0$
Area = 0

Thus,
 $F_x - F_y = 0$
 $F_x = F_y$

Similarly, In Y-dir

$-F_y (2x \cdot 2z) \sin \theta + F_x (2x \cdot 2z) \sin \theta - W = 0$

$F_y = F_x$

Thus,

$F_x = F_y = F_z$

Pressure at any pt in a fluid at rest has same magnitude in all directions. In other words,

when a certain pressure is applied at any point in a fluid at rest, the pressure is equally transmitted in all the directions and to every other point in the fluid.

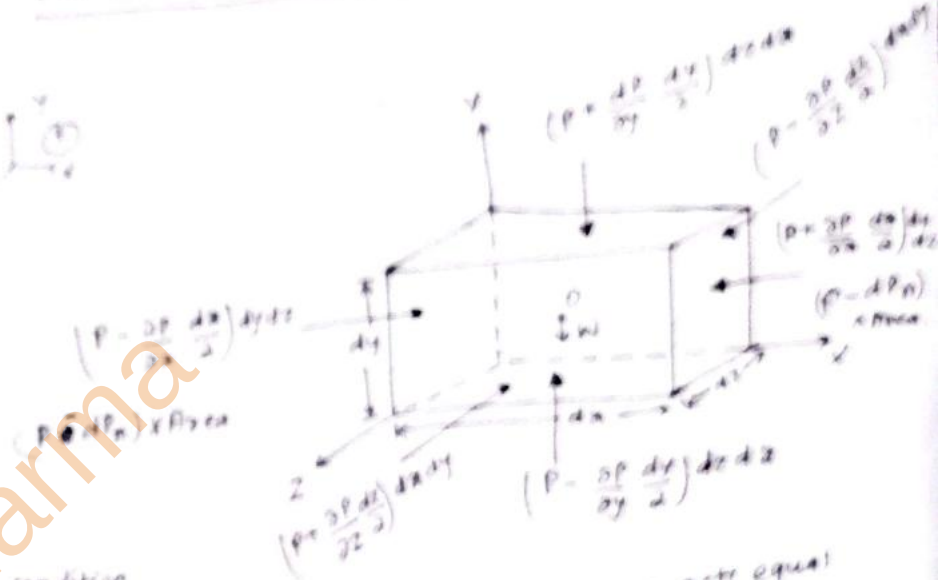
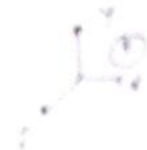
Pascal's law

Partially law
 Applied for point loads and static equilibrium only

Product of these infinitesimal quantities
 $W = \gamma \text{ Volume}$
 $= \gamma x \frac{1}{2} (2x \cdot 2y \cdot 2z)$

Product of these infinitesimal quantities
 are neglected compared to other terms.
 Neglect $\rightarrow$ due to small size of element

Hydraulic law of pressure distribution
 (or) Pressure depth relationship



In static condition,
 $\sum F_x = 0$

$(P - \frac{\partial P}{\partial x} \frac{dx}{2}) dy dz - (P + \frac{\partial P}{\partial x} \frac{dx}{2}) dy dz = 0$

or $\frac{\partial P}{\partial x} = 0$ { Vol $\neq 0$ }

In z-dir

$(P - \frac{\partial P}{\partial z} \frac{dz}{2}) dx dy - (P + \frac{\partial P}{\partial z} \frac{dz}{2}) dx dy = 0$

or $\frac{\partial P}{\partial z} = 0$

In y-dir

$(P - \frac{\partial P}{\partial y} \frac{dy}{2}) dz dx - (P + \frac{\partial P}{\partial y} \frac{dy}{2}) dz dx - W = 0$

Pressure force acts equal & opposite in all direction

Change in pressure,

$dP = \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy + \frac{\partial P}{\partial z} dz$
 $P = f(x, y, z)$

Change in pressure along x-dir
 Change in pressure along z-dir

1/2 is done so that change in pressure is same along both dir (Pascal's law)

Thus, Total change = dP

$$\sum F_x = 0$$

$$-P_s (\partial s \partial z) \sin \theta + P_x (\partial y \partial z) = 0$$

$$\text{or, } -P_s (\partial s \partial z) \frac{\partial y}{\partial s} + P_x \partial y \partial z = 0$$

$$\text{or, } (P_x - P_s) \underbrace{\partial y \partial z}_{\text{Area} \neq 0} = 0$$

Thus,

$$P_x - P_s = 0$$

$$\boxed{P_x = P_s}$$

Similarly, In Y-dir:

$$-P_s (\partial s \partial z) \cos \theta + P_y (\partial z \partial x) - W = 0$$

$$\boxed{P_y = P_s}$$

Thus,

$$\boxed{P_x = P_y = P_s}$$

∴ Pressure at any pt in a fluid at rest has same magnitude in all directions. In other words, when a certain pressure is applied at any point in a fluid at rest, the pressure is equally transmitted in all the directions and to every other point in the fluid.

→ Pascal's law

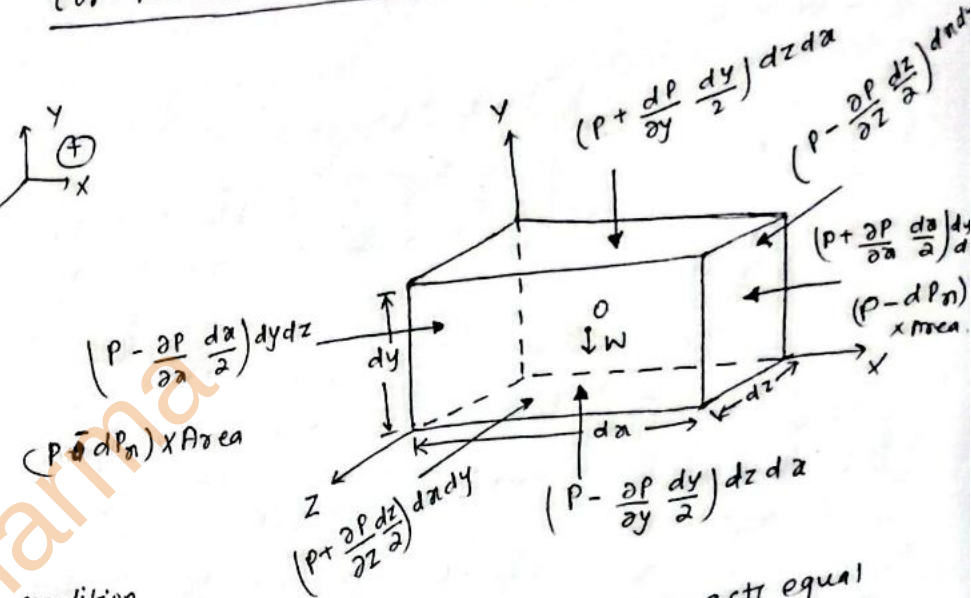
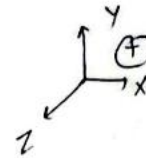
Pascal's Law
Applied for point loads and static equilibrium only

{ wt of element
 $W = \gamma \cdot \text{Volume}$
 $= \gamma \times \frac{1}{2} (\partial x \partial y \partial z)$

Product of three infinitesimal quantities hence neglected compared to other two.

Neglect → doesn't mean zero contribution

• Hydraulic law of pressure distribution
 (or) Pressure depth relationship



In static condition,

$$\sum F_x = 0$$

$$\text{or, } (P - \frac{\partial P}{\partial x} \frac{dx}{2}) dy dz - (P + \frac{\partial P}{\partial x} \frac{dx}{2}) dy dz = 0$$

$$\text{or, } \frac{\partial P}{\partial x} = 0 \quad \{ \text{vol} \neq 0 \}$$

In z-dir,

$$(P - \frac{\partial P}{\partial z} \frac{dz}{2}) dx dy - (P + \frac{\partial P}{\partial z} \frac{dz}{2}) dx dy = 0$$

$$\text{or, } \boxed{\frac{\partial P}{\partial z} = 0}$$

In y-dir,

$$\text{or, } \boxed{\frac{\partial P}{\partial y} = -\gamma} \quad (P - \frac{\partial P}{\partial y} \frac{dy}{2}) dz dx - (P + \frac{\partial P}{\partial y} \frac{dy}{2}) dz dx - W = 0$$

$$\gamma (\partial x \partial y \partial z)$$

∴ pressure force acts equal & opposite in all direction

∴ change in pressure,

$$dP = \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy + \frac{\partial P}{\partial z} dz$$

$$P = f(x, y, z)$$

change in pressure along x-dir

change in pressure along z-dir

∴ $\frac{1}{2}$ is done so that change in pressure is same along both dir (Pascal's law)

Thus, Total change = dP_x

∴ Pressure intensity (P) at any point in a static mass of fluid does not vary in 'x' and 'z' direction and it varies only in 'y' direction.

Along x & z dir. $\frac{\partial P}{\partial x} = 0$ & $\frac{\partial P}{\partial z} = 0$ point at pressure same everywhere.

∴ First, partial derivative $\frac{\partial P}{\partial y}$ may be reduced to total (or) derivative as follows

∴ $\frac{dP}{dy} = -\gamma$ → P along y at any change $\frac{y}{\gamma} = \frac{P}{\gamma}$ to.

→ minus sign in above equation signifies that the pressure decreases in the direction in which 'y' increases (i.e. in upward direction)

Eqⁿ (2) is the basic differential equation representing the variation of pressure in a fluid at rest, which holds for both compressible and incompressible fluids.

Eqⁿ (2) indicates that within a body of fluid at rest, the pressure increases in downward direction at the rate equivalent to the specific weight (γ) of the liquid.

Further, if $dy = 0$, then 'dp' is also equal to 0 which means that pressure remains constant over any horizontal plane in a fluid.

∴ Pressure at any point in an incompressible fluid is constant and hence integrating the eqⁿ (2) we get.

$$P = -\gamma y + C \quad (3)$$

Boundary condⁿ,

Pressure at free surface (P) = Atmospheric pressure (P_{atm})

& Depth of fluid (y) = 0

Eqⁿ (3) becomes,

$$P_{atm} = 0 + C$$

$$\therefore C = P_{atm}$$

Thus, eqⁿ (3) becomes,

$$P = -\gamma y + P_{atm}$$

Pressure intensity at any point lying below the datum in any mass of liquid is calculated by substituting the values of 'y' and ' P_{atm} '. Here, 'y' = -h (since point lying below the datum) and

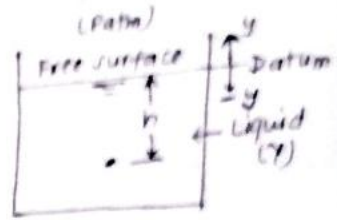
' $P_{atm} = 0$ ' (since assuming the gauge pressure),

$$\therefore P = \gamma h$$

$h = \frac{P}{\gamma}$, vertical ht of free surface above any point in a liquid at rest (Pressure head)

Pressure at any point in a mass of liquid → depends only upon vertical depth below free surface

↳ sp. wt. of liquid
↳ doesn't depend on shape and size of bounding container.



∴ Datum at top → '-'
Datum at bottom → '+'

✓ Pressure at any pt in a compressible fluid

For compressible fluid,

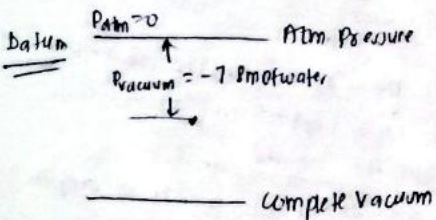
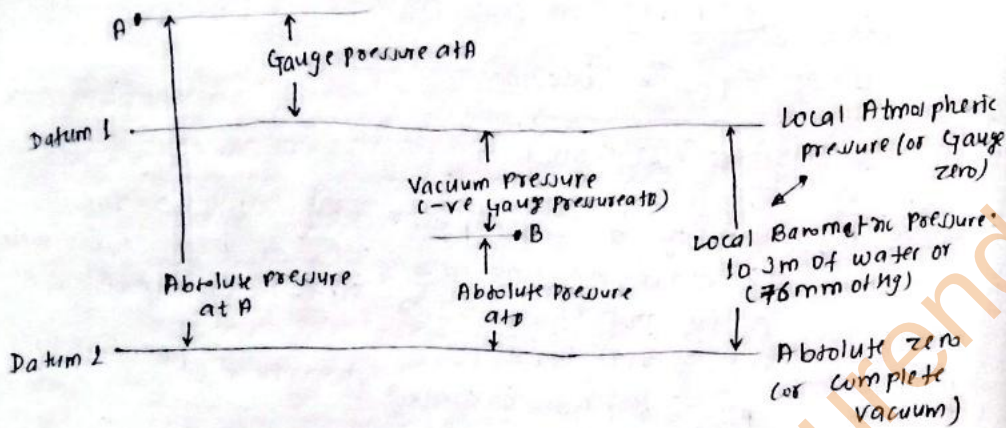
- ↳ density varies with pressure
- ↳ No free surface

If P_1 & P_2 are pressure intensities at two pts which are at elevation y_1 and y_2 above arbitrarily assumed datum

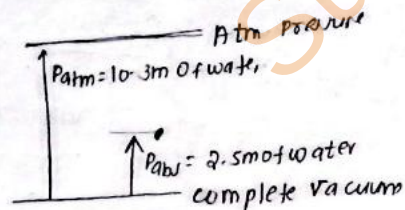
Integrating eqⁿ *

$$\int_{P_1}^{P_2} \frac{dP}{\gamma} = - \int_{y_1}^{y_2} dy = y_1 - y_2 \quad \text{--- (**)}$$

• Atmospheric, Absolute, Gauge and Vacuum Pressures



(i) Working in terms of vacuum pressure



(ii) Working in terms of Absolute pressure

$$P_{\text{absolute}} = P_{\text{atm}} \pm P_{\text{gauge}}$$

Acc to Question, Find the reference level

At sea level, under normal conditions the equivalent values of the atmospheric pressure are $1.01043 \times 10^5 \text{ N/m}^2$; or $1.03 \text{ kg (f) / cm}^2$; or 10.3 m of water ; or $760 \text{ mm of mer. cury}$.

Pressure head $\left(\frac{P}{\gamma} \right) h$

$$P = \gamma h$$

For water,

$$P = \gamma_w \times h_w = 9810 \times 10.3 = 1.01 \times 10^5 \text{ N/m}^2$$

$$P = \gamma_{Hg} \times h_{Hg} = (13.6 \times 9810) \times (760 \times 10^{-3}) = 1.01 \times 10^5 \text{ N/m}^2$$

$$1.03 \text{ kg (f) / cm}^2 = \frac{1.03 \times 9.81}{10^{-4}} = 1.01 \times 10^5 \text{ N/m}^2$$

Vacuum $\rightarrow$ -ve pressure

Boiling of water starts at

-7.8m of water
or, 7.8m of water, vacuum
or, 2.5m of water absolute.

Vacuum pressure
or
negative gauge pressure
or
suction pressure

• Measurement of Pressure

- Tire pressure and blood pressure are gauge pressure by convention
- Atm. pressure, deep vacuum pressure and altimeter pressures must be absolute
- pressure are often expressed as a depth of particular fluid (e.g. meter of water)
- common choice: mercury (Hg) & water

$$P = \gamma h$$

• various devices measuring fluid pressures

- Manometers
- Mechanical Gauges

(I) Manometers

- ↳ pressure measuring device
- ↳ Based on principle of balancing the column of liquid (whose pressure is to be found)

(a) Simple Manometers

- ↳ measure pressure at a point in a fluid contained in a pipe or a vessel

(b) Differential manometers

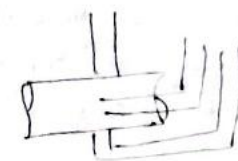
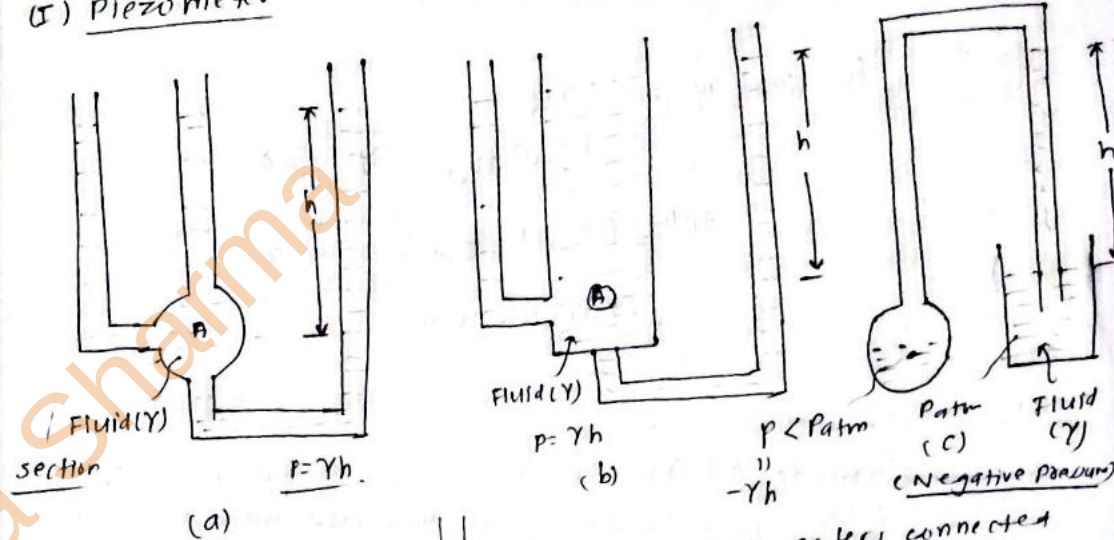
- ↳ measure diff. of pressure b/w any two points in a fluid contained in a pipe or a vessel.

• Simple Manometers

- Piezometer
- U-tube Manometer
- Single Column Manometer

Consists glass tube with one end connected to gauge point where pressure is to be measured and other remain open to atmosphere

(I) Piezometer



3 - piezometers connected
↓
However all measure same pressure.

- ↳ Simplest form which measures moderate pressures.
- ↳ tube extends vertically upward st ht that liquid can freely rise into without overflowing.
- ↳ pressure at any pt in the liquid is indicated by ht of liquid in the tube above that pt.
- ↳ measures only gauge pressure, since surface of liquid in tube subjected to atmospheric pressure.

ie $P_{atm} = 0$

$P = \gamma h$ where $\gamma \rightarrow$ sp. unit wt. of liquid
 $h \rightarrow$ 'pressure head at 'A'

→ All burrs and surface roughness near the hole to be removed and better to round the edge of hole slightly

→ Hole should be small, not larger than 3mm

→ To prevent capillary action, internal diameter less than 1mm should not be used

For capillary rise taken,

$$h = h_{\text{gauge}} - \text{Capillary rise}$$

Capillary fall, $h = h_{\text{gauge}} + \text{Capillary fall}$

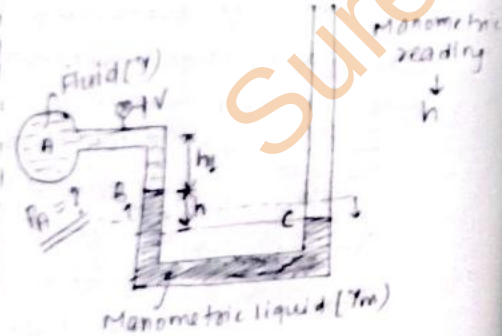
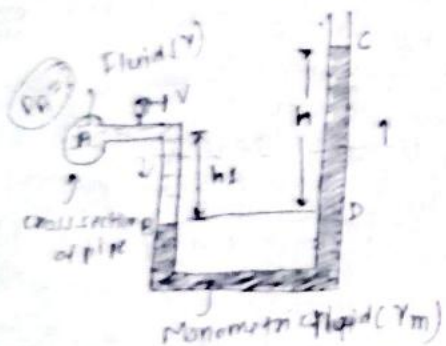
Lower meniscus

(II) U-tube Manometer

→ Piezometers cannot be used when large pressures in the lighter liquids are to be measured, since this would require very long tube, which cannot be handled conveniently

→ gas pressures cannot be measured as gas forms no free surface

→ These limitations overcome in U-tube manometer



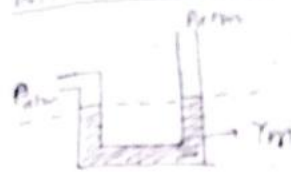
(a) U-tube Simple Manometer (Measuring +ve Pressure)

(b) Measuring -ve pressure

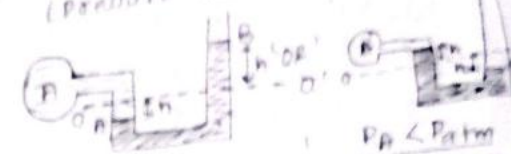
→ $\gamma_m > \gamma$

→ Pressure at section A, $P_A = ?$

Without connecting



After connecting to Gauge Pt. A (Pressure measuring Pt.)



NOTE

Manometric height (h)/reading (h)

→ Use static manometric liquid with pressure gauge to connect static level at difference (Fig (a) & (b))

$P_A > P_{atm}$
Fluid (Y) of Manometric liquid (γ_m) at static level

$P_A < P_{atm}$
Fluid (Y) will be raised by γ_m i.e. Manometric fluid rises towards A by ht. h from initial level O-O (Succ.) fluid

→ V. Air relief valve (dissolved air/gases in fluid is released before measuring pressure)
h: Manometric reading 'h'

If $\gamma_m < \gamma$

→ Fluid of Manometric liquid at static level

→ Sometimes more than one liquid can be used in manometer. The liquids used in manometer should not get mixed with fluids of which the pressure are to be measured.

→ Choice of manometric liquid depends upon range of pressure to be measured.

→ For low pressure range, liquids of lower specific gravity are used. For high pressure range, generally mercury is employed. Manometric liquids: mercury, oil, salt solution, carbon tetrachloride, bromoform, alcohol, water (But should be immiscible with fluid)

$$P = \gamma_m h$$

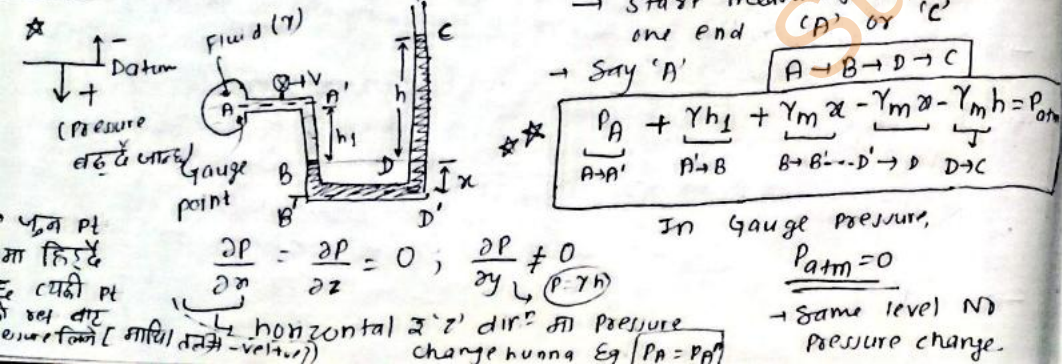
General procedure may be adopted to obtain gauge equation.

(i) Start from either 'A' or from free surface in the open end of manometer and write the pressure there in an appropriate unit (say 'm' of water or other fluid or N/m^2 or $kg(f)/cm^2$ or $kg(f)/m^2$). If pressure is unknown (as at 'A') it may be expressed in terms of appropriate symbol on the other hand, the pressure at free surface in the open end (which is equal to atmospheric pressure) may be taken as zero. So, that eqⁿ formed in each case will represent gauge pressure.

(ii) To the pressure found above, add the change in pressure which will be caused while proceeding from one level to another adjacent level of contact of liquids of diff specific gravities. Use positive sign if next level of contact is lower than first and negative if it is higher.

(iii) Continue the process as in (ii) until other end of gauge is reached and equate the expⁿ to the pressure at that point known or unknown. The expression will contain only one unknown (i.e. pressure at A), which may thus be evaluated.

Concept:



→ Same fluid in tube and manometer (Here, Pt B & D को बीचको Pressure लेबल पढेन)
जति तल जाओ त्यही माथि जान्छे, सो, Being at B mean (same fall & rise in same fluid) we reached D is same

For -ve pressure (Fig b)

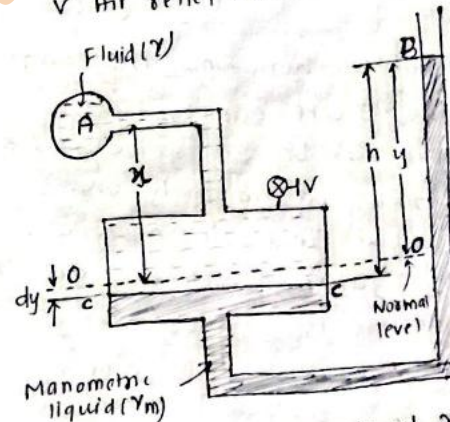
$$P_A + \gamma h_1 + \gamma_m h = P_{atm} = 0$$

(III) Single Column Manometer

U-tube Manometer: (i) Readings at two or more points (B & C) are to be taken for fluid levels.
(ii) Errors in readings simplified as more readings taken.

→ Modified U-tube manometer in which a shallow reservoir having a large cross-sectional area 'A' (about 100 times) as compared to area of tube 'a'.

h' Manometric reading
V Air relief valve



→ Before joining to fluid gamma, Normal level (OO) → original level
a) Vertical Single Column Manometer

(b) Inclined Single Column Manometer

(Inclined माथि small pressure change पनि distinctly measure गर्न सकिन्छ) → More sensitive

Gauge eqⁿ, Starting from A,

$$P_A + \gamma x + \gamma dy - \gamma_m h = 0$$

$$\text{or, } P_A + \gamma x + \gamma dy - \gamma_m (y + dy) = 0$$

$$\text{or, } P_A = \gamma_m y - \gamma x + dy(\gamma_m - \gamma) \quad \text{--- (E)}$$

But, (Volume)_{fall} = (Volume)_{rise}

$$\text{or, } dy \cdot A = y \cdot a$$

$$\text{or, } dy = \frac{y \cdot a}{A}$$

For sufficiently large, $A \gg a$ [$A \approx 100a$]
reservoir
So, dy can be neglected.

$$P_A = \gamma_m y - \gamma x \quad \text{--- (F)}$$

→ Thus, only one reading of height of level of liquid in narrow tube (i.e. y) is required to be taken to obtain pressure head at 'A' while value of 'x' is known.

→ However if ' dy ' is appreciable, then since the terms within bracket of eqⁿ (E) are constant, the scale on which ' y ' is read can be so graduated as to correct for ' dy ' so that again only one reading of height of liquid level in narrow tube is required to be taken, which will directly give pressure head at 'A'.

→ For inclined manometer,

$$V_{\text{fall}} = V_{\text{rise}} \quad \text{inclined ht.}$$

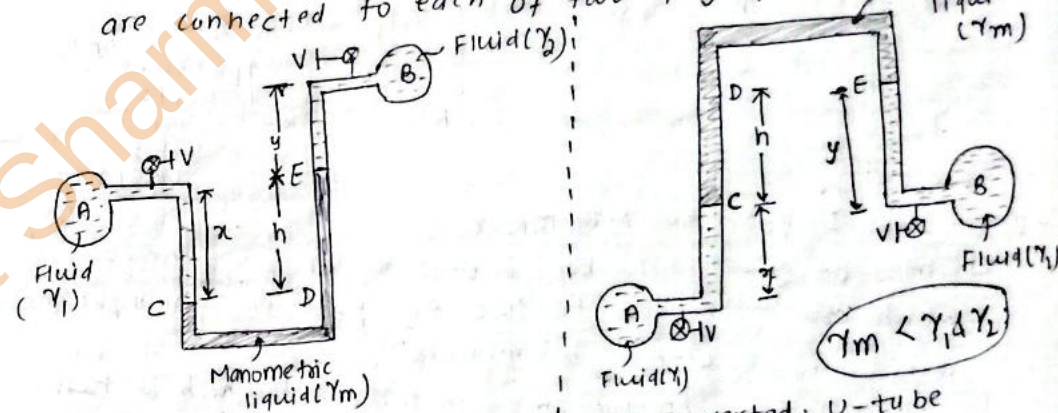
$$\text{or, } A(dy) = a(l)$$

$$\therefore dy = \left(\frac{a}{A}\right)l = \left(\frac{a}{A}\right) \frac{y}{\sin \theta}$$

→ Single column manometers can also be employed to measure negative Gauge pressures. If the pressure at 'A' in container is negative, the manometric liquid surface in the reservoir will be raised by a certain distance and consequently there will be drop in liquid surface of tube.
→ Since no reading is required to be taken for the level of liquid in reservoir, it need not be made of transparent material.

⊗ Differential Manometers

↳ Measures difference of pressure b/w any two points
↳ consists of a bent glass tube the two ends of which are connected to each of two gauge points.



(a) U-tube Differential Manometer

Gauge eqⁿ, starting from A,

$$P_A + \gamma_1 x - \gamma_m h - \gamma_2 y = P_B$$

$$P_A - P_B = \gamma_m h + \gamma_2 y - \gamma_1 x$$

(b) Inverted U-tube Differential Manometer

Gauge eqⁿ,

$$P_A - \gamma_1 x - \gamma_m h + \gamma_2 y = P_B$$

$$P_A - P_B = \gamma_1 x + \gamma_m h - \gamma_2 y$$

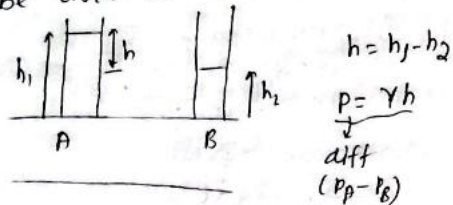
Common types:

- (i) Two-Piezometer manometer
- (ii) U-tube Differential Manometer
- (iii) Inverted U-tube Differential Manometer

(I) Two-Piezometer Manometer

↳ consists of two separate piezometers which are inserted at the two gauge points b/w which the diff. of pressure is required to be measured.

↳ useful only when pressure at each of two points is small. Moreover, it cannot be used to measure the pressure difference in gases.



(II) U-tube Differential Manometer

↳ consists of glass tube bent in U-shape, the two ends of which are connected to two gauge points b/w which pressure diff. is required to be measured.

↳ lower part contains manometric liquid which is heavier than liquid which pressure difference is to be measured and is immiscible with it.

Gauge eqn

Fig(a) Pressure diff, $p_A - p_B = \gamma_m h + \gamma_2 y - \gamma_1 x$

↳ However for small pressure diff., mercury makes precise measurement difficult, and hence for such case it is common to use a liquid which is only slightly heavier than the liquid for which pressure difference is to be measured.

(III) Inverted U-tube Differential Manometer

↳ U shaped and held inverted

↳ suitable for measurement of small pressure difference in liquids. The upper parts of this manometer is filled with a manometric liquid which is lighter than liquid for which pressure diff. is to be measured and is immiscible with it.

↳ In general, air is used as manometric fluid, which is easy to get compressed. {light manometric fluid used}

Gauge eqn

Fig(b), $p_A - p_B = \gamma_1 x + \gamma_m h - \gamma_2 y$

↳ Increasing the sensitivity $\Rightarrow$ ① Specific unit wt. of manometric fluid and fluid at 'A' & 'B' nearly equal to each other, then large values of 'h' obtained for small pressure differences.

② inclining the gage tubes:

vertical gage diff. 'h' is transposed into a reading which is magnified by $(\frac{1}{\sin \theta})$

$\theta \rightarrow$ inclination with horizontal.

Mechanical Gauges

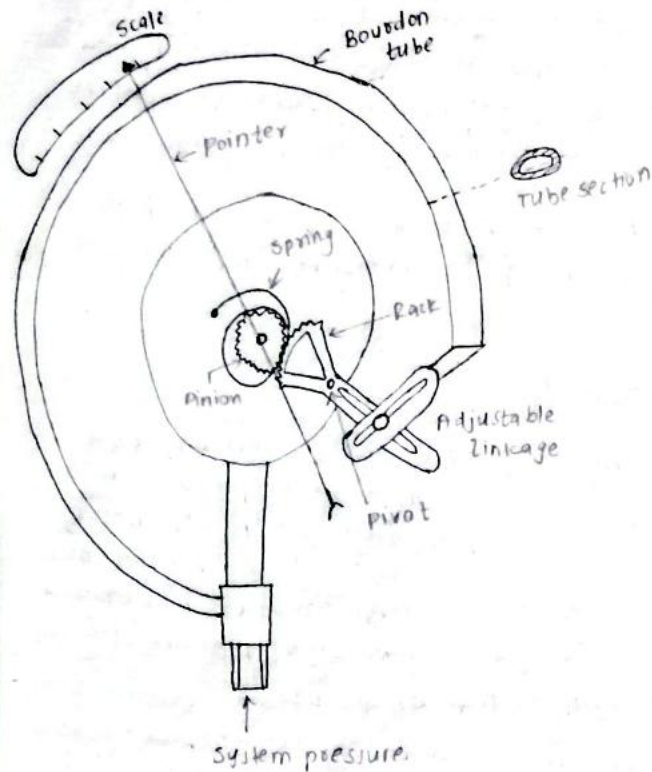


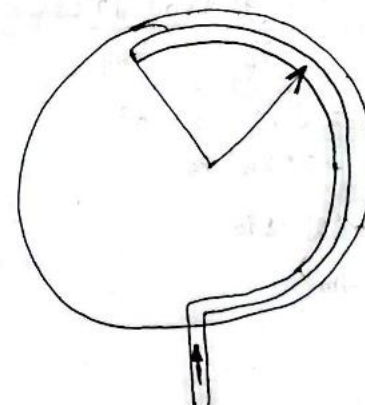
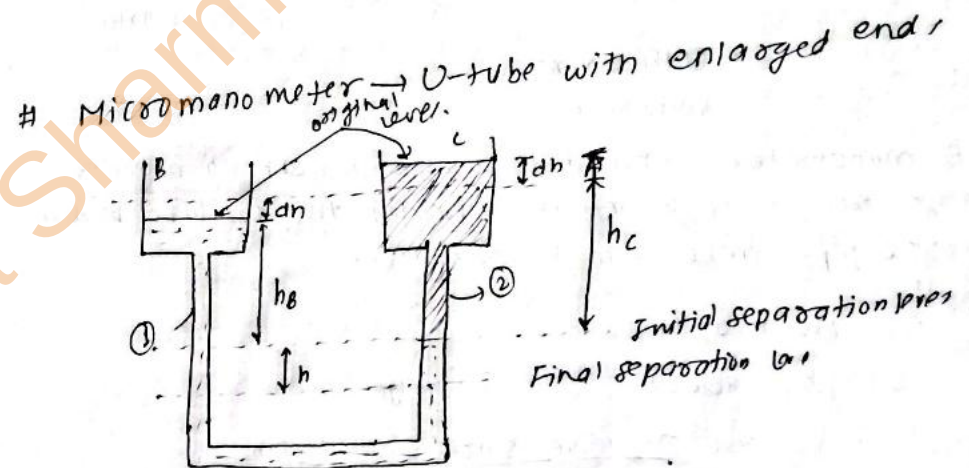
Fig. Bourdon Tube Pressure Gauge
(most common type invented by E. Bourdon)

- ↳ pressure measuring device
- ↳ embody an elastic ~~member~~ element, which deflects under the action of the applied pressure, and this movement mechanically magnified, ~~and~~ operates a pointer moving against a graduated circumferential scale.
- ↳ measuring high pressure, and where high precision is not required
- ↳ pressure responsive element in this gauge is tube of steel or bronze which is of elliptic cross section and is curved into a circular arc.

- ↳ The tube is closed at its outer end, and the end of the tube is free to move. The other end of tube, through which fluid enters, is rigidly fixed to the frame.
- ↳ When gauge is connected to gauge point, fluid under pressure enters the tube. Due to increase in internal pressure, the elliptical cross-section of the tube tends to become circular, thus causing tube to straighten out slightly.
- ⇒ The small outward movement of the free end of tube is transmitted through a link, quadrant and pinion, to a pointer which by moving clockwise on graduated circular dial. Indicates pressure intensity of fluid.

- The dial of gauge is so calibrated that it reads pressure inside tube equals local atm pressure.
- ↳ When a vacuum gauge is connected to a partial vacuum, tube tends to close, thereby moving the pointer in anti-clock direction indicating negative or vacuum pressure.
- ↳ Gauge dials are usually calibrated to read (N/m^2) , or (Pa) , or $(kg(f)/cm^2)$. However other units of pressure, such as meters of water or centimeters of mercury, are also frequently used.

(-ve pressure, sucks the air inside Bourdon tube)



Numericals

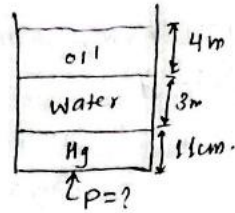
① Determine the pressure at the bottom of an open tank of it contains layers of 4m of oil (9.016 kN/m^3), 3m of water and 11cm of Hg.

→ Sol:

$$\gamma_{\text{oil}} = 9.016 \text{ kN/m}^3$$

$$\gamma_{\text{water}} = 9.81 \text{ kN/m}^3$$

$$\gamma_{\text{Hg}} = 13.61 \times 9.81 \text{ kN/m}^3$$



$$P_{\text{bottom}} = P_{\text{oil}} + P_{\text{water}} + P_{\text{Hg}}$$

$$= (9.016 \times 1000 \times 4) + (9810 \times 3) + (13.61 \times 9810) \times 11 \times 10^{-2}$$

$$= 80180.5 \text{ N/m}^2$$

$$= 80.18 \text{ kN/m}^2$$

② A monometer connected to a pipe indicates a negative gauge pressure of 60mm of Hg. What is the absolute pressure in the pipe? Take atm. pressure = 1 Bar

→ Sol:

We know,

$$P_{\text{absolute}} = P_{\text{atm}} \pm P_{\text{gauge}}$$

For negative gauge, take '-' sign

$$P_{\text{gauge}} = \frac{(13.61 \times 9810 \times 60 \times 10^{-3})}{\gamma_{\text{Hg}}}$$

$$= 8010.846 \text{ N/m}^2$$

$$\text{Thus, } 1 \text{ Bar} = 1.01 \times 10^5 \text{ Pa}$$

$$P_{\text{absolute}} = 1.01 \times 10^5 - 8010.846$$

$$= 91.199 \text{ kN/m}^2$$

③ What force 'F' is required for equilibrium? Neglect wt. of A. Take area(A) = 38.7 cm^2 , Area(B) = 387 cm^2 and wt.(B) = 4500N

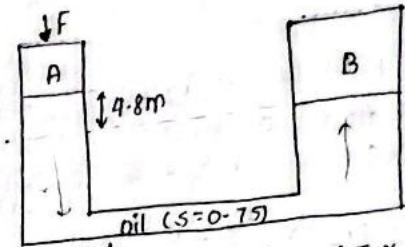
→ Sol:

Starting from end A,
Pressure eq. becomes

$$\frac{F}{A_A} + \gamma_{\text{oil}} \times h_{\text{oil}} - \frac{W_B}{A_B} = 0$$

$$\text{or, } \frac{F}{38.7 \times 10^{-4}} + (0.75 \times 9810 \times 4.8) - \frac{4500}{387 \times 10^{-4}} = 0$$

$$\text{or, } F = 313.33 \text{ N}$$



Follow one Rule (For many num.)
Write pressure eq. from one side
 $P = \frac{F}{A} = \gamma h$

④ Find the specific gravity of olive oil from figure, if absolute pressure at bottom is 240 kPa.

→ Sol:

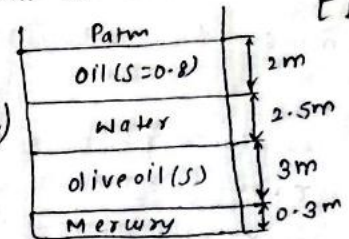
$$P_{\text{bottom}} = P_{\text{atm}} + (P_{\text{oil}} + P_{\text{water}} + P_{\text{olive}} + P_{\text{Hg}})$$

$$\text{or, } 240 \times 10^3 = (0.8 \times 9810 \times 2) + (9810 \times 2.5)$$

$$+ (S \times 9810 \times 3)$$

$$+ (13.61 \times 9810 \times 0.3) + P_{\text{atm}}$$

$$\Rightarrow S = 1.99$$



Absolute $\rightarrow P_{\text{atm}} = 1.01 \times 10^5 \text{ N/m}^2$
Gauge $\rightarrow P_{\text{atm}} = 0 \text{ N/m}^2$

- ⑤ Find pressure at 'A' of a liquid of specific gravity 1.36 rises 2.5m in the tube? (-24.95 kN/m²)

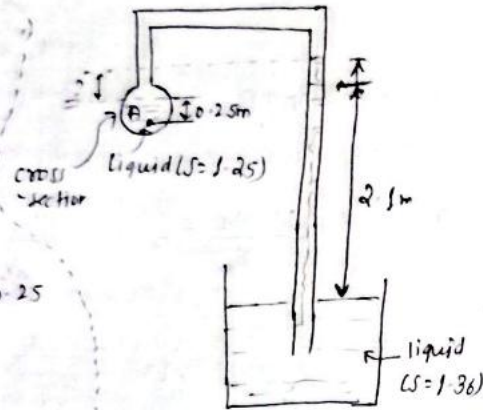
→ Solⁿ:

Writing pressure equation starting from B liquid (S=1.36)

$$-(1.36 \times 9810) \times 2.5 + 0 = P_A$$

$$P_A = -24951.735 \text{ N/m}^2$$

$$P_A = -24.95 \text{ kN/m}^2$$



* खाली space → there is air so, its γ is small, so can be neglected

- ⑥ Determine the reading of gauge 'A' from the figure.

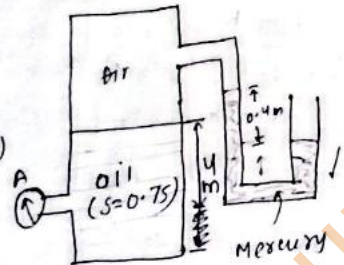
→ Solⁿ:

Writing pressure eq: starting from mercury end

$$-(13.6 \times 9810 \times 0.4) + (0.75 \times 9810 \times 4) = P_A$$

$$P_A = -23936.4 \text{ N/m}^2$$

$$P_A = -23.94 \text{ kPa}$$



{ P_{air} = 0 }

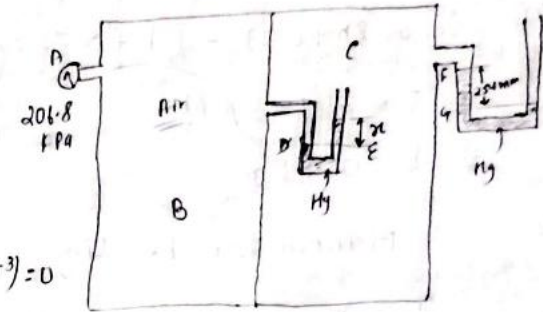
- ⑦ In the figure, compartment 'B' and 'C' are closed and filled with air. What would be value of 'x'? (1.8m)

→ Solⁿ:

Writing pressure eq: starting from A

$$206.8 - (13.6 \times 981) \times x + (13.6 \times 981) \times 2.5 \times 10^{-3} = 0$$

$$x = 0.180 \text{ m}$$



Given data are: [Air & γ neglect जिन कारणों]
तरफ → +ve
माफिर → -ve

- ⑧ The reading of the manometer given in the figure shows when the vessel is empty. Find the manometer reading when vessel is completely filled with water. (0.425m)

→ Solⁿ:

Writing pressure eq: starting from end of Mercury

$$(13.6 \times 9810 \times 0.2) - (9810 \times y) - (9810 \times 2) = 0$$

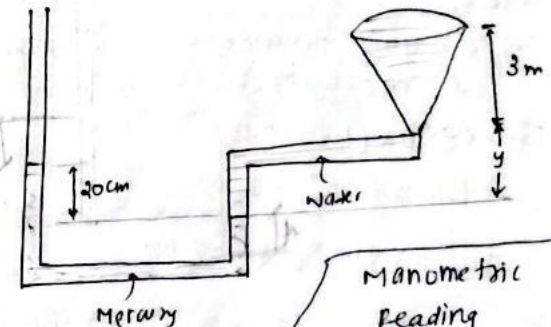
⇒ y =

- ⑩ When vessel is empty, (the level of mercury is as shown)

$$(13.6 \times 9810 \times 0.2) - (9810 \times y) = 0$$

$$\Rightarrow y = 2.72 \text{ m}$$

- ⑪ When vessel is filled with water (level of mercury changes)
2nd case → level show (h) manometric reading changes



Manometric Reading
↓
Manometric fluid
के level के Difference

$$(13.6 \times 9810) \times (h + 0.2 + h) - (9810) \times (h + y + 3) = 0$$

$$\Rightarrow (13.6 \times 9810)$$

$$\text{or, } 13.6(2h + 0.2) - (h + 5.72) = 0$$

$$\text{or, } -3 = h - 27.2h$$

$$\Rightarrow h = 0.115 \text{ m}$$

$$\therefore \text{Manometric Reading} = h + h + 0.2 = 0.429 \text{ m}$$

Q. For a compound manometer shown in figure below, what is gage pressure at 'A' if the manometric fluid is mercury and the fluid in the pipe and in the tubing which connects the two V-tubes is water? (3.9 kg(f)/cm^3)

→ Sol:

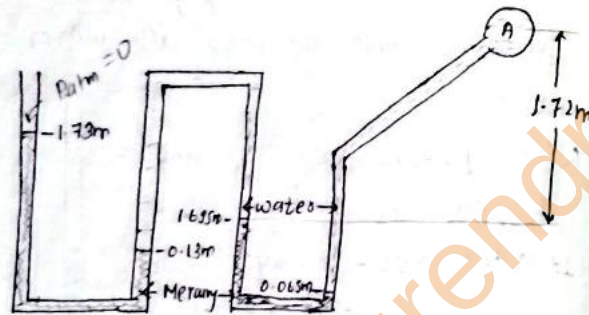
Given

Starting from end of mercury,

$$13.6 \times 9810 \times (1.73 - 0.13) - 9810 \times (1.695 - 0.13) + 13.6 \times 9810 \times (1.695 - 0.065)$$

$$- 9810 \times (1.695 - 0.065 + 1.72) = P_A$$

$$\text{or, } P_A = 382712.53 \text{ N/m}^2 = 39013 \text{ kgf/m}^2 = 3.9 \text{ kg(f)/cm}^2$$



$$1 \text{ kg} = 9.81 \text{ N}$$

10. Calculate the pressure at 'B' from figure below, if diameter reservoir and tube are 50mm and 5mm respectively. (-0.49 kPa)

→ Sol:

Here,

$$V_{\text{fall}} = V_{\text{rise}}$$

small tube reservoir

$$0.25 \times a = dy \times A$$

$$\text{or, } 0.25 \times \frac{\pi}{4} \times (5 \times 10^{-3})^2 = dy \times \frac{\pi}{4} \times (50 \times 10^{-3})^2 \quad (S=0.8)$$

$$\text{or, } dy = 2.5 \times 10^{-3} \text{ m}$$

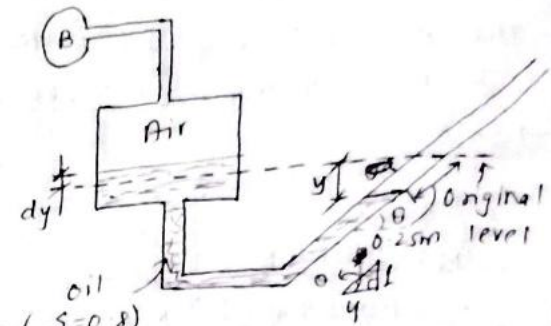
Now,

Writing pressure eq. starting from B,

$$P_B + \gamma_{\text{oil}} \times dy + \gamma_{\text{oil}} \times y = 0$$

$$\text{or, } P_B + 0.8 \times 9810 \times (2.5 \times 10^{-3} + 0.0606) = 0$$

$$\Rightarrow P_B = -495.5 \text{ Pa} = -0.496 \text{ kPa}$$



$$y = \frac{0.25 \times \sin \theta}{\sin(\tan^{-1} \frac{1}{4})} = 0.2425 \text{ m}$$

$$0.0606$$

$$0.2425$$

Q1) As shown in the accompanying figure, pipe 'M' contains carbon tetrachloride of sp. gravity 1.594 under a pressure of 1.05 kg(f)/cm^2 and 'N' contains oil of sp. gravity 0.8. If the pressure in the pipe 'N' is 1.75 kg(f)/cm^2 and the manometric fluid is mercury, find the difference 'x' b/w the levels of mercury. (14.25 cm)

→ Solⁿ:

Here,

$$P_M = 1.05 \text{ kg(f)/cm}^2 \\ = 1.05 \times 9.81 \times 10^4 \text{ N/m}^2 \\ = 103005 \text{ N/m}^2$$

$$P_N = 1.75 \text{ kg(f)/cm}^2 \\ = 1.75 \times 9.81 \times 10^4 \text{ N/m}^2 \\ = 171675 \text{ N/m}^2$$

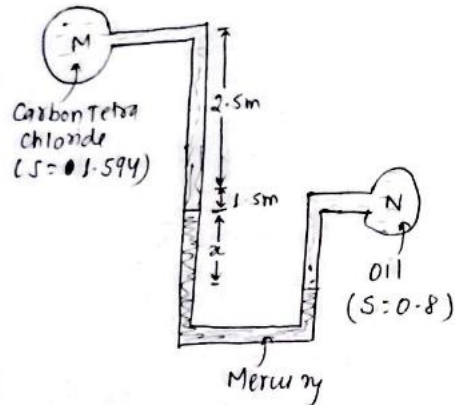
Now,

Writing pressure equation starting from M,

$$P_M + (1.594 \times 9810) \times (2.5 + 1.5) + 136 \times 9810 \times x - 0.8 \times 9810 \times (x + 1.5) = P_N$$

$$\text{or, } 103005 + 62548.56 + 135903 + 125568x = 0$$

$$\text{or, } \frac{-5650.56}{125568} - 17893.44 = -125568x \\ \Rightarrow x = 0.1425 \text{ m} \\ x = 14.25 \text{ cm}$$



Q2) Two pipes as shown in figure convey toluene of sp. gravity 0.875 and water respectively. Both the liquids in the pipes are under pressure. The pipes are connected to a U-tube manometer & the hoses connecting the pipes to the tubes are filled with corresponding liquids. Find the difference of pressure in two pipes if the level of manometric liquid having a specific gravity 1.25 is 2.25m higher in the right limb than lower of toluene in the left limb of manometer. (7.4 kN/m²)

→ Solⁿ:

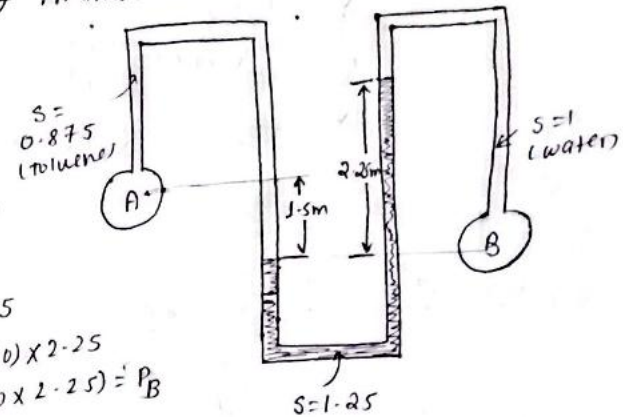
$$P_A - P_B = ?$$

Writing pressure equation, starting from A,

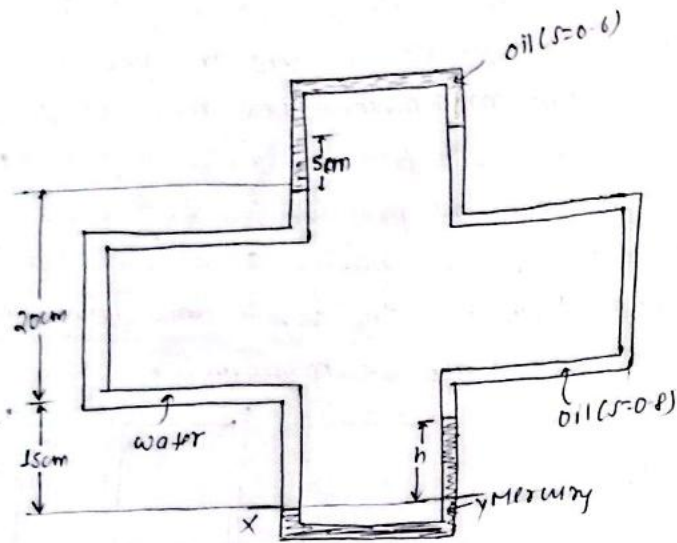
$$P_A + (0.875 \times 9810) \times 1.5 - (1.25 \times 9810) \times 2.25 + (9810 \times 2.25) = P_B$$

$$\Rightarrow P_A + 07357.5 = P_B$$

$$P_B - P_A = 7.36 \text{ kN/m}^2$$



(13) From the figure below, find the value of 'h'.



→ Solⁿ.
Writing

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial z} = 0$$

$$\frac{\partial P}{\partial y} = -\gamma$$

$$P_x = P_y$$

$$[9810 \times 35 \times 10^{-2}] + [(0.6 \times 9810) \times 5 \times 10^{-2}] = (0.8 \times 9810) \times (40 \times 10^{-2} - h) + (13.6 \times 9810) \times h$$

$$\Rightarrow h = 4.69 \times 10^{-3} \text{ m}$$

$$h = 0.47 \text{ cm}$$

- # Points to be considered for pressure comp.
- Any surface open to atm. has atm. press. of gage pressure, i.e., taken zero.
 - The shape of container doesn't matter in value of pressure as it is independent on head and density and not on wt. of fluid.
 - Pressure at same depth in a continuous mass of fluid are equal.

$$P_{\text{next level}} = P_{\text{prev. level}} + \gamma h$$

[h → +ve down ward, -ve up wards]

- For diff. fluids, press. is continuous at interface.
- In case of inclined part, take eqv. vert. ht. of fluid to compute pressure at a point.
- Press. due to air in closed container is negligible.

Effect of capillarity

For capillary rise: true static ht. = given ht. of fluid - cap. rise

For capillary fall: true static ht. = given ht. of fluid + cap. fall.

Chapter-4

Hydrostatic Forces on Surfaces

- Total pressure & center of pressure
- When a static mass of fluid comes in contact with surface either plane or curved, a force is exerted by the fluid on the surface. This force is known as "Total pressure (or Resultant Force or Pressure force; F)".
- Since for fluid at rest no tangential force exists, total pressure acts in the direction normal to the surface. The point of application of total pressure on surface is called 'Center of pressure (CP)'.
- Engineer: compute magnitude of total pressure and locate its point of application to design hydraulic structures.

✓ Total pressure or Resultant force (F) = Pressure intensity (P) * Area of plane (A)

Considering force on each element of area (dA), then

F = Sum of forces on all the elements of plane surface

✓ $\therefore F = \sum dF = \sum p dA$ — (I)

Valid if the dirⁿ of pressure on each element is parallel to each element.

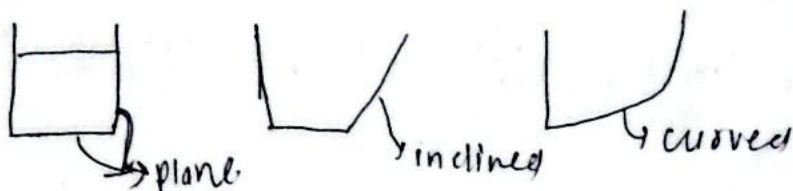
p → pressure intensity on element
 dA → Area of element.

All dF s must act parallel to each other on area.

press. Intensity. $\uparrow$
 $\downarrow$
 $F \equiv$ Pressure force $\equiv$ Resultant force $\equiv$ total pressure $= P \times A$

Pressure Intensity (P) = γh
free surface to point

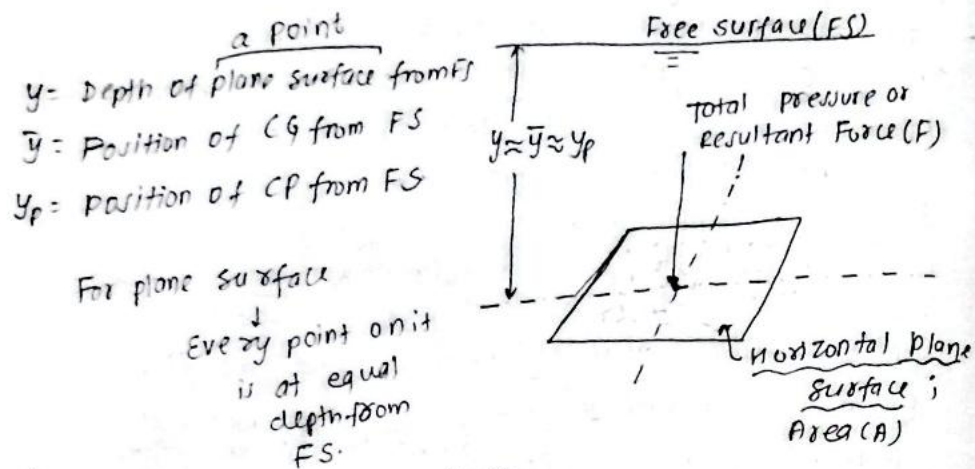
Normally act at right Angle



CG → centroid of given surface area
Centre of gravity
whole wt. pass thru pt.

Total pressure on a plane surface.

(1) Total pressure on a Horizontal plane surface.



Pr Intensity: $P = \gamma y$ (Uniform) $\rightarrow$ same
So, Total pressure (F) passed th. CG.

$\rightarrow$ Negligible thickness

✓ Total pressure or Resultant force (F):

Since every point on the surface is at the same depth below the free surface of the liquid, the pressure intensity (p) is constant over the entire plane surface.

If 'A' is total area of the surface, then the total pressure force (or resultant force) is

$$F = \sum dF = \sum p dA = p \sum dA = pA$$

$$F = (\gamma y) A \quad \because \text{pressure intensity } (p) = \gamma y$$

$$\therefore F = \gamma A y \approx \gamma A \bar{y}$$

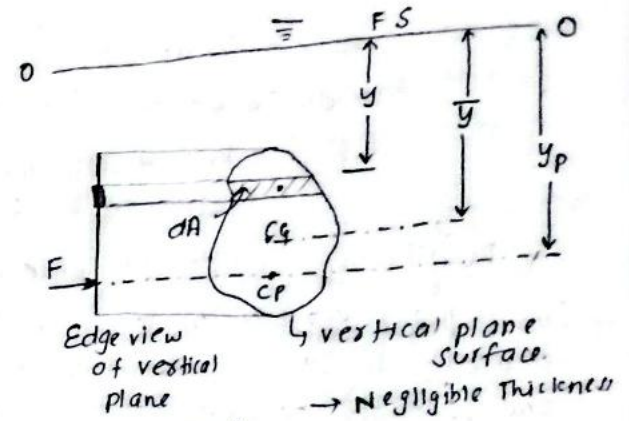
Here, pressure intensity (p) is uniform over the surface.

✓ Position of center of pressure (y_p):

For a plane surface immersed horizontally since the pressure intensity is uniform, the total pressure would pass through the centroid of the area, i.e., in this case the centroid of the area and the center of pressure (CP) coincides with each other.

(2) Total pressure on a vertical plane surface :-

$\rightarrow$ Pressure intensity (p) changes at each pt as depth is different
 $\downarrow$
So take elementary area and integrate.
 $\rightarrow$ Incompressible fluid
 $\downarrow$
 γ const



✓ Total pressure or Resultant force (F)

Since, depth of liquid varies from point to point, on the surface, the pressure intensity is not constant over the entire surface. As such the total pressure on the surface may be determined by dividing entire surface into a number of small parallel strips and computing total pressure on each of these strips.

A summation of these total pressures on the small strips will give the total pressure on entire plane surface.

$\rightarrow$ horizontal strip of area (dA) at vertical depth y below FS.
 $\therefore$ thickness of strip is very small, for this strip the pressure intensity (p) may be assumed to be constant.
Total pressure force on the strip,

$$dF = p dA = (\gamma y) dA \quad \text{--- (iii)}$$

Total pressure force (or resultant force) on entire vertical plane

$$F = \sum dF = \sum (\gamma y) dA$$

$$F = \gamma \sum y dA \quad \left\{ \text{or } \gamma \int y dA \right\}$$

First moment of area

$$F = \gamma \bar{y} A$$

$$\therefore \boxed{F = \gamma A \bar{y}} \quad \text{--- (I)}$$

where, $\sum y dA = \bar{y} A$ represents first moment of areas of the strip about an axis through 'O-O'.

$\gamma \bar{y}$ → Pressure intensity at centroid of surface area (i.e. (C)).

✓ Position of center of pressure (y_p):

→ For a plane surface immersed horizontally, since the pressure intensity is uniform the total pressure would pass through the centroid of area (C.G.), i.e. in this case the centroid of the area and the center of pressure (C.P.) coincides with each other. However, for a plane surface immersed vertically the center of pressure does not coincide with centroid of area. Since, pressure intensity increases with increase in depth of liquid, the center of pressure (C.P.) lies below centroid of surface area (C.G.).

→ Total moment of the total pressure (F) on a whole vertical plane surface about an axis thr. O-O' (i.e. about free surface) is

$$\rightarrow M = F y_p \quad \text{--- (V)}$$

Now, Total moment of total pressure (dF) on small strip about an axis (O-O') is

$$\rightarrow dM = dF y = \gamma y^2 dA$$

Sum of moments of the total pressure on all strips,

$$\rightarrow M = \sum dM = \sum \gamma y^2 dA = \gamma \sum y^2 dA$$

$$\therefore M = \gamma I_0 \quad \text{--- (VI)}$$

→ where, $\sum y^2 dA$ = sum of second moment of areas of strips about an axis through O-O' which is equal to moment of inertia (I_0) of plane surface about an axis O-O'.

By using "Principle of Moments", which states that, "The moment of resultant of a system of forces about an axis is equal to the sum of moments of the components about the same axis. So, equating

(V) & (VI)

$$\rightarrow F y_p = \gamma I_0$$

or, $(\gamma A \bar{y}) y_p = \gamma I_0$

$$\therefore \boxed{y_p = \frac{I_0}{A \bar{y}}} \quad \text{--- (VII)}$$

Further from "Parallel Axis Theorem",

$$I_0 = I_G + A \bar{y}^2 \quad \text{--- (VIII)}$$

I_G → MOI about axis passing through centroid of area and parallel to axis O-O'

$$\{ \because F = \gamma A \bar{y} \text{ from (I)} \}$$

$$M = \sum dM$$

$$\downarrow$$

$$F y_p = \sum \gamma y^2 dA$$

$$\text{or, } \gamma A \bar{y} y_p = \gamma I_0$$

$$\text{or, } y_p = \frac{I_0}{A \bar{y}}$$

Thus,

$$y_p A \bar{y} = I_G + A \bar{y}^2$$

$$\text{or } y_p = \frac{I_G}{A\bar{y}} + \frac{A\bar{y}^2}{A\bar{y}}$$

$$y_p = \bar{y} + \frac{I_G}{A\bar{y}} \quad (IX)$$

→ Since, for any plane surface the factor $\frac{I_g}{A \bar{y}^2}$ is always positive, eqⁿ (IX) indicates, that

' $y_p > \bar{y}$ ' (i.e. center of pressure (CP) is always below the centroid of Area (CG))

✓ → Deeper the surface submerged i.e. greater the value of 'y', the factor $\frac{I_G}{Ay}$ becomes smaller

and C_P comes closer to C_G of plane surface.

and CP comes closer to the surface.

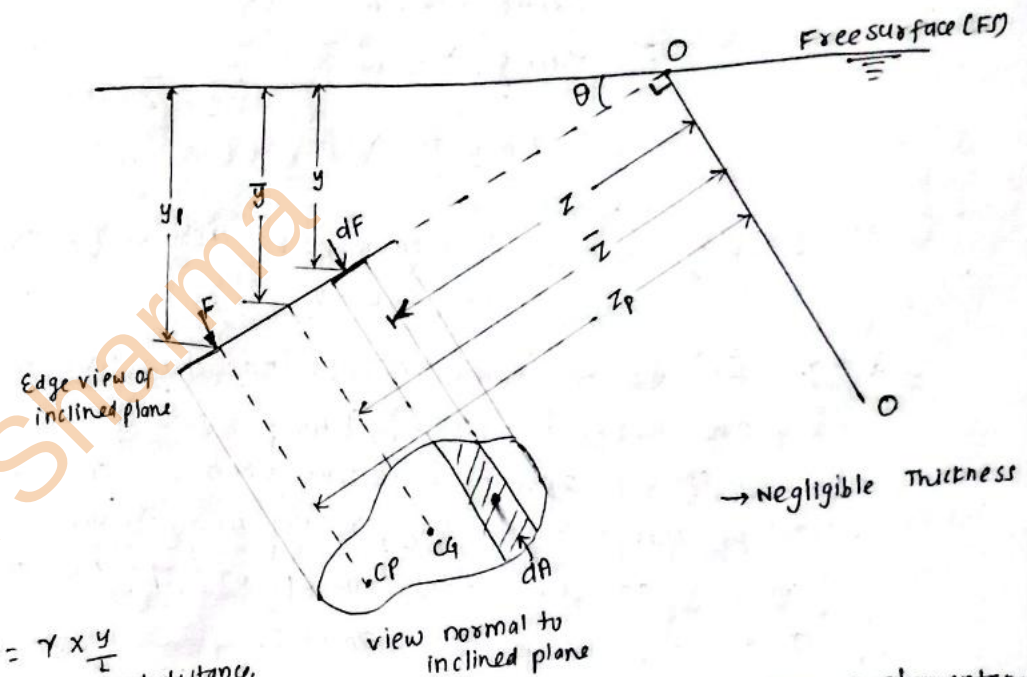
Q (As the pressure becomes greater with increasing depth, its variation over a given area becomes smaller in proportion, thereby making the distribution of pressure more

uniform)
 $\Rightarrow$ Thus, where $\frac{dP}{dz}$ is negligible variation of pressure is negligible at the CP may be taken as approximately at the centroid. This is justifiable in liquids, only if the depth is very large and the area is small, and in gases because in them pressure changes very little with depth. (or negligible for gas)

$$\begin{aligned}\bar{y} &\rightarrow \text{FS to CG} \\ y_p &\rightarrow \text{FS to CP}\end{aligned}$$

$$y_p \rightarrow FS \text{ to } CP$$

(3) Total pressure on an inclined plane surface.



$$P = \gamma \times \frac{y}{L}$$

vertical distance.

vertical distance, inclined plane
 $\propto$ Diff depths for diff. points so, integration of elementary areas req.

↳ Depth always vertical

- ✓ Depth always vertical
- ✓ Area $\times$ normal distance from XY } $\rightarrow$ Area moment.

→ For strip, pressure intensity can be assumed constant. Hence
total pressure force on the strip is

$$dF = p dA = (\gamma y) dA = \gamma (Z \sin \theta) dA \quad \text{--- (x)}$$

So, Total pressure (or resultant force) on entire inclined plane surface is

$$F = \sum dF = \sum \gamma (Z \sin \theta) dA$$

$$F = \gamma \sin \theta \sum Z dA$$

$$F = \gamma \sin \theta (\bar{Z} A)$$

$$\{\bar{y} = \bar{Z} \sin \theta\}$$

$$\boxed{F = \gamma A \bar{y}} \quad \text{--- (XI)}$$

$$\sum Z dA = \bar{Z} A \rightarrow \text{first moment of area of the strip about } O-O'$$

⇒ Eqⁿ (IV) & (XI) are same thereby indicating that for a plane surface wholly submerged in a static mass of liquid and held either vertical or inclined, total pressure is equal to product of pressure intensity at the centroid of area (i.e. $p = \gamma \bar{y}$) and area of plane surface (A)

✓ Position of center of pressure (y_p):-

Moment of resultant force (F) on whole inclined plane surface about axis through O-O' is

$$M = F Z_p \quad \text{--- (XII)}$$

Total moment of total pressure (dF) on small strip about an axis O-O' is

$$dM = dF Z \quad \left\{ \because dF = p dA = (\gamma y) dA \right.$$

$$dM = \gamma \sin \theta Z^2 dA \quad \left. = \gamma (Z \sin \theta) dA \right\}$$

The sum of moments of total pressures on all the strips becomes

$$M = \sum dM = \sum \gamma \sin \theta Z^2 dA = \gamma \sin \theta \sum Z^2 dA$$

$$M = (\gamma I_0) \sin \theta \quad \text{--- (XIII)}$$

$\sum Z^2 dA$ = second moment of area of the strips about an axis thr. O-O' which is equal to MOI (I_0), of plane surface about O-O'.

Using "Principle of Moments",

$$F Z_p = (\gamma I_0) \sin \theta$$

$$[\because F = \gamma A \bar{y}]$$

$$Z_p = \left(\frac{I_0}{A \bar{y}} \right) \sin \theta \quad \text{--- (XIV)}$$

From, "Parallel Axis Theorem",

$$I_0 = I_G + A \bar{Z}^2 \quad \text{--- (XV)}$$

I_G → MOI of area about an axis through centroid of area and || to axis O-O'.

Thus, From (XIV) & (XV),

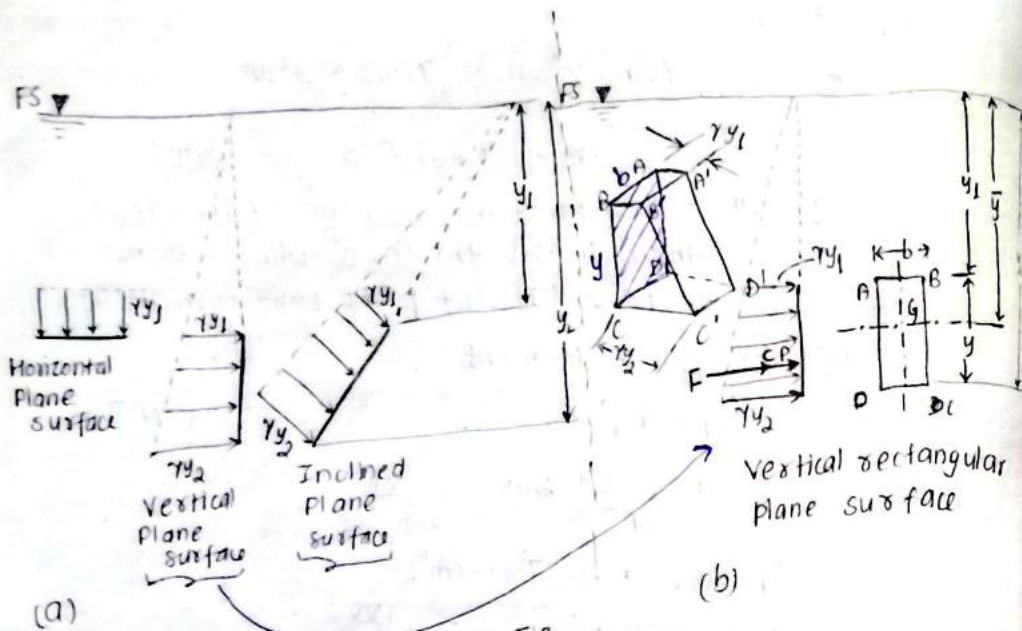
$$\frac{Z_p (A \bar{y})}{\sin \theta} = I_G + A \bar{Z}^2$$

$$\text{or, } \frac{y_p (A \bar{y})}{\sin^2 \theta} = I_G + \frac{A \bar{y}^2}{\sin^2 \theta} \quad \left\{ \because \bar{y} = \bar{Z} \sin \theta \right. \\ \left. y_p = Z_p \sin \theta \right\}$$

$$\text{or, } y_p = \frac{I_G \left(\frac{\sin^2 \theta}{A \bar{y}} \right) + \frac{A \bar{y}^2}{\sin^2 \theta} \left(\frac{\sin^2 \theta}{A \bar{y}} \right)}$$

$$\boxed{y_p = \bar{y} + \frac{I_G \sin^2 \theta}{A \bar{y}}} \quad \text{--- (XVI)}$$

Pressure diagram for different plane surfaces:



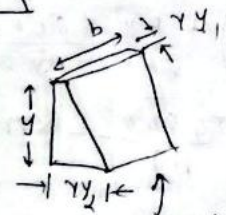
pressure intensity, $p = \gamma y$ — w
 $\rightarrow F = \gamma A \bar{y} = \gamma (by) \left(y_1 + \frac{y_2 - y_1}{2} \right)$

→ Trapezoidal pressure diagram

$\{ A = by ; y = \frac{y_2 - y_1}{2} \}$
 $\boxed{DF = \gamma by \left(\frac{y_1 + y_2}{2} \right)}$ — w

A → Area of ABCD

Area of trapezoid = $\left(\frac{a+b}{2} \right) h$

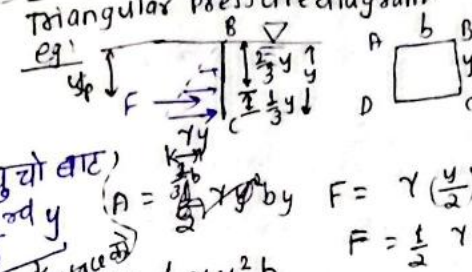


Volume of prism
 $= A_{trap} \times b$
 $= \frac{1}{2} (\gamma y_1 + \gamma y_2) \times y \times b$

Trapezoidal section vol. = $\gamma by \left(\frac{y_1 + y_2}{2} \right)$

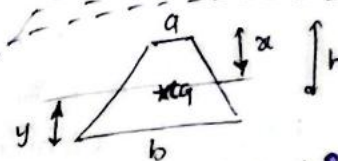
Thus, Total pressure on plane Surface (F) = Volume of prism.

Triangular pressure diagram



$y_p = \bar{y} + \frac{I_G}{A \bar{y}} = \frac{y}{2} + \frac{by^3}{12(by) \frac{y}{2}}$

$\boxed{y_p = \frac{y}{2} + \frac{y}{3} = \frac{2y}{3}}$



$x = \frac{h}{3} \left[\frac{a + 2b}{a + b} \right]$

$y = \frac{h}{3} \left[\frac{2a + b}{a + b} \right]$

→ जहाँ बाएँ लिखें
 जो चपको opposite
 x 2 जगह

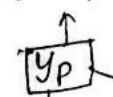
$y_p = \bar{y} + \frac{I_G}{A \bar{y}}$

where, $\bar{y} = y_1 + \frac{y}{2} = y_1 + \frac{y_2 - y_1}{2} = \frac{y_1 + y_2}{2}$

For ABCD, $I_G = \frac{by^3}{12}$ { $y = y_2 - y_1$ }
 $A = by$

Thus,
 $y_p = \left(\frac{y_1 + y_2}{2} \right) + \frac{\frac{b(y_2 - y_1)^3}{12}}{by \times \frac{y_1 + y_2}{2}}$

Trapezoidal Pressure Diagram को CG



→ Trapezoidal section को CG
 → ABCD plane section को CP.

Or, $y_p = y_1 + \frac{y}{2} + \frac{y^2}{12 \left(\frac{y_1 + y_2}{2} \right)}$

$y_p = y_1 + \frac{y}{2} + \frac{y(y_2 - y_1)}{6(y_1 + y_2)}$

$y_p = y_1 + \frac{y(3y_1 + 3y_2 + y_2 - y_1)}{6(y_1 + y_2)}$

$\boxed{y_p = y_1 + \frac{y(y_1 + 2y_2)}{3(y_1 + y_2)}}$

★ ★
 ← position of CP.

Total pressure on a curved plane surface (VV Imp)

- 'ABC' is the trace of curved surface which extends in the direction normal to the plane of paper
- At any point on curved surface also the pressure acts normal to the surface
- 'dA' area of small element of curved surface lying at vertical depth (y) below free surface (FS) of liquid, then total pressure (i.e. resultant force) acting normal to elementary surface area is

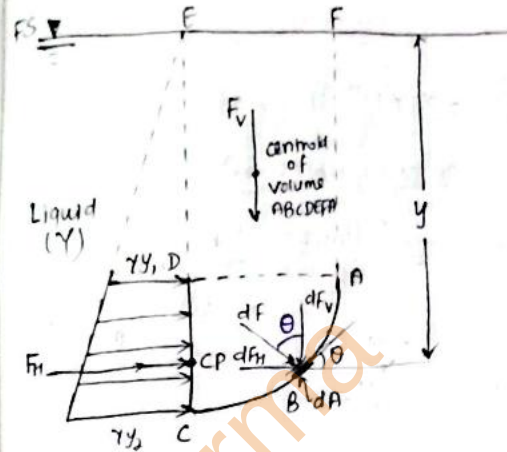
$$dF = p dA = (\gamma y) dA \quad \text{--- (XX)}$$

Total pressure (or Resultant Force) on curved surface.

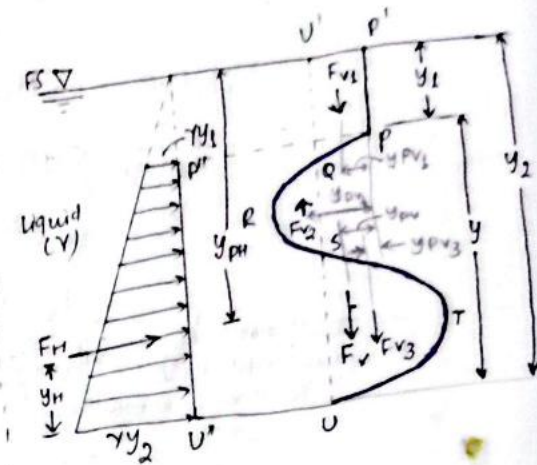
$$F = \sum dF = \sum p dA = \sum (\gamma y) dA \quad \text{--- (XXI)}$$

However, in case of curved surface, the direction of total pressure (or resultant force) on elementary area varies from point to point. As such the determination of total pressure on a curved surface from equation (XXI) is rather impossible.

Simplified, by resolving total pressure (F), on curved surface into horizontal and vertical components, ' F_H ' and ' F_V '.



(a)



(b)

Fig. Total pressure on curved surface

→ depth changes ⇒ Pressure intensity changes ⇒ so, take elementary strip.

(a) elementary area dA

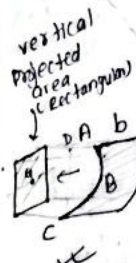
Every component force is resolved into vertical & horizontal component. & added to calculate total force.

$$dF_H = dF \sin \theta = (\gamma y) dA \sin \theta$$

$$F_H = \sum dF_H$$

$$F_H = \gamma \sum y dA \sin \theta$$

(vertical projection of dA)



= $\gamma \sum y dA'$ First moment of area

$$F_H = \gamma A' \bar{y}$$

(Rectangular projected area)

A → Overall ABC की Vertical में Project जहाँ की Area.
 $\bar{y} = ED + \frac{CD}{2}$ { Vertical में Project जहाँ की Area की CG }

all dF ||
 all dF ||
 all dF ||
 So, $F = \sum dF$
 curved surface not ||
 So, can't be added directly

θ = Inclination of elementary area with horizontal

Similarly, For vertical,

$$F_v = \sum dF_v = \sum dF \cos \theta = \sum (\gamma y) dA \cos \theta$$

$$F_v = \gamma \sum (y dA \cos \theta)$$

$$F_v = \gamma \sum dV$$

Total Pressure on Horizontal projection $F_v = \gamma V$
Total Vertical Force

(गो लंबाई)
आदि लंबाई liquid
को wt

$F_v \rightarrow$ passes thr ABCDEFA volume की centroid

$F_h \rightarrow$ Passes thr centroid of trapezoidal area

Resultant, $F = \sqrt{F_h^2 + F_v^2}$
 $\theta = \tan^{-1} \left(\frac{F_v}{F_h} \right)$

Fig(b)

(1) Use where the pressure force is asked to be computed
 $\rightarrow$ Find force on PQRSU
(or may ask P'PQRSU)

Project in vertical plane

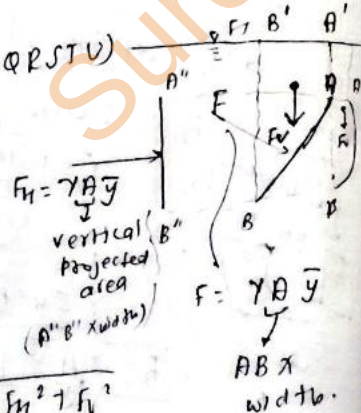
★ end pts

$$F_h = \gamma A \bar{y}$$

$\bar{y}$ = $y_1 + \frac{y}{2}$ $y_h = \frac{y}{3} \left(\frac{2y_1 + y_2}{y_1 + y_2} \right)$

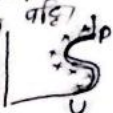
Passes thr. CP (centroid of trap)

$$F = \sqrt{F_h^2 + F_v^2}$$



$$y_{ph} = y_1 + \frac{y}{3} \left(\frac{y_1 + 2y_2}{y_1 + y_2} \right) = y_2 - y_h$$

Vertical force

- (i) Boundary pts $\rightarrow$ P & U
- (ii) अंशको mark जग (along wetted part) (Blue pen) 
- (iii) Boundary pts लाई head zero बनाउने

$P \rightarrow P'$, $U \rightarrow U'$
vertically free surface मा प्रतिबिम्ब

- (iv) Find each areas (components)
 $U'P'PQ$; QRS ; STU
 $Vol^m = \sum Area \times width(b)$
 $Wt = \sum (Vol^m \times \gamma)$
 $\rightarrow W_1 = Fv_1$; $W_2 = Fv_2$; $W_3 = Fv_3$
 $\rightarrow F_v = \sum Fv_i$ (with sign)

- (v) Find CG of each components from some pt (say 'P')

$$Fv_1 \rightarrow \gamma p v_1$$

$$Fv_2 \rightarrow \gamma p v_2$$

$$Fv_3 \rightarrow \gamma p v_3$$

- (vi) evaluate centroid of whole area
Calculated either by
• taking moment abt line PP' (vertical line)

$$F_v \times y_{pv} = \sum Fv_i \times y_{p v_i} = Fv_1 \times y_{p v_1} + Fv_2 \times y_{p v_2} + Fv_3 \times y_{p v_3}$$

• OR abt. any pt P (consider F_h)

Thus, calculated $y_{pv} = \dots$

★ Note: Marked जबके तिर Area पयो → Force downward

Marked जबके Opposite तिर Area पयो → Vertically upward.

FS not given.

↳ convert to corresponding head.

$$P = \gamma h$$

$$10 \times 10^3 = \gamma h$$

calculate h,

P को corresponding head,

$$h = 1.019 \text{ m}$$

अति गहिर जाने

→ -ve Pressure → Surface अन्तर तल जाने
→ that corr. FS is called
↓
imaginary fluid line

Numericals.

(1)

Side

$$P = \gamma A \bar{y}$$

$$= 9.81 \times (2.5 \times 2) \times (4 + \frac{2}{2})$$

$$= 245.25 \text{ kN}$$

Passes through y_p .

$$y_p = \bar{y} + \frac{I_G}{A \bar{y}}$$

$$= 5 + \frac{6 \times 5 / (2.5 \times 2^3)}{12}$$

$$= 5.07 \text{ m from E.}$$

(2)

Bottom

$$(b) P_{\text{bottom}} = P_{BC} = \gamma A \bar{y}$$

$$= 9.81 \times (6.5 \times 2.5) \times (4 + 2)$$

$$P_{BC} = 956.475 \text{ kN}$$

$$(c) \text{ Total wt} = \gamma \times \text{Vol}^m$$

$$= 9.81 \times (6.5 \times 2 \times 2.5 + 0.01 \times 4)$$

$$= 319.217 \text{ kN}$$

Diff b/w (b) & (c)

$$P_{AD} = \gamma A \bar{y}$$

$$= 9.81 \times (6.5 \times 2.5) \times 4$$

$$= 637.25 \text{ kN}$$

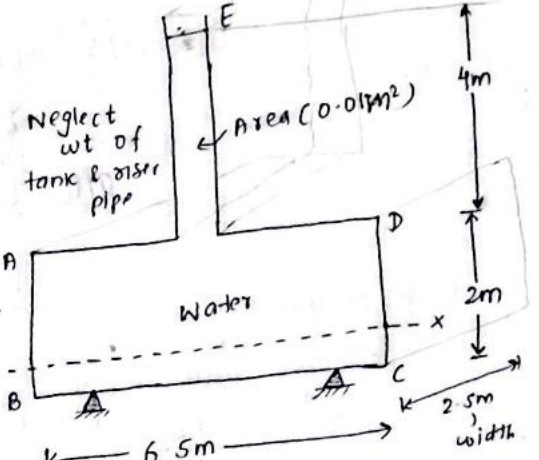
वेनिग Hole

of A = 0.01 m²

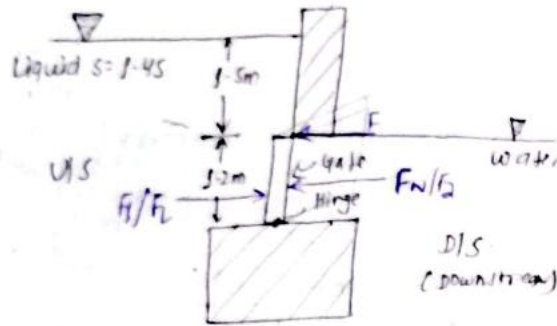
Resultant vertical force,

$$= P_{AD} + P_{BC}$$

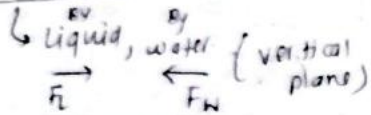
$$= 956.475 \text{ kN}$$



Gate side view



Forces on Gate,



thickness not given
assume negligible

∑ Moment at Hinge = 0

$$F_1 = \gamma A \bar{y} \quad (1.2 \times 2.4)$$

$$= (1.45 \times 9.81) \times 1.2 \times (1.5 + \frac{1.2}{2})$$

$$F_1 = 71.69 \text{ kN} (\rightarrow)$$

$$F_2 = \gamma A \bar{y}$$

$$= 9.81 \times 2.4 \times (0.6)$$

$$= 14.13 \text{ kN}$$

$$(\leftarrow)$$

$$\text{Resultant force } F_R = F_1 - F_2$$

$$= 57.56 \text{ kN}$$

Locate:

$$y_{p1} = \bar{y} + \frac{I_G}{A \bar{y}}$$

$$= 2.1 + \frac{(2 \times 1.2^3)}{12}$$

$$= 2.16 \text{ m. from (L.S.)}$$

$$y_{p1} = \bar{y} + \frac{I_G}{A \bar{y}}$$

$$= 0.6 + \frac{(2 \times 1.2^3)}{12}$$

$$= 0.8 \text{ m}$$

Varignon's theorem, (from bottom)

$$F_R \times y_p = F_1 \times (2.7 - y_{p1}) + F_2 (1.2 - y_{p2})$$

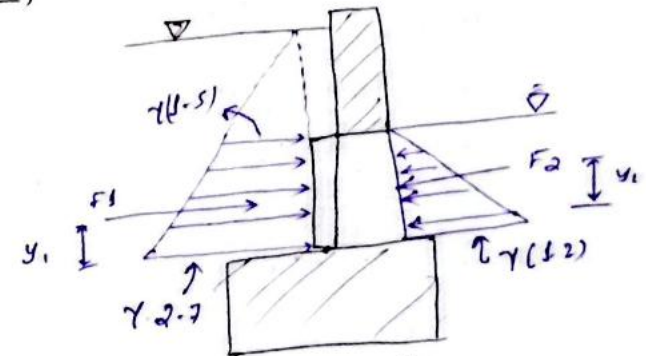
$$y_p = 0.574 \text{ m (from hinge)}$$

We know, ∑ M about Hinge = 0 (∑ +ve)

$$\text{or, } F_{\text{top}} \times 1.2 - F_R \times 0.574 = 0$$

$$\text{or, } F_{\text{top}} = F = 27.53 \text{ kN} (\leftarrow)$$

Next method, (Pressure diagram)



$$\text{Force } (F_1) = \text{vol. of pressure dia.}$$

$$= \text{Area of trap} \times \text{width.}$$

$$= \left\{ \frac{1}{2} [\gamma(1.5) + \gamma(2.7)] \times 1.2 \right\} \times 2$$

$$= 71.69 \text{ kN} (\rightarrow)$$

$$F_2 = \text{Volum. of press. dia.}$$

$$P_{\text{trap}} = \frac{1}{2} (a+b)h = \left\{ \frac{1}{2} \times \frac{(1 \times 9.81) \times 1.2}{\cancel{9.81}} \right\} \times \cancel{2} \times 1.2$$

$$= 14.13 \text{ kN} (\leftarrow)$$

$$F_R = F_1 - F_2 = 57.56 \text{ kN} (\rightarrow)$$

Location:

$$y_1 = (\bar{y}) \text{ from bottom/hing}$$

$$= \frac{1.2}{3} \left[\frac{2 \times 1.2 + 2.7}{1.2 + 2.7} \right]$$

$$= 0.523 \text{ m}$$

$$y_2 = (\bar{y}) \text{ from bottom}$$

$$= \frac{1.2}{3} \times 1.2$$

$$= 0.4 \text{ m}$$

③ Solⁿ:

★ { side view: rectangular $\rightarrow$ pressure diagram
 ↳ other $\rightarrow$ Formula

Total pressure (F)

$$= \gamma A \bar{y}$$

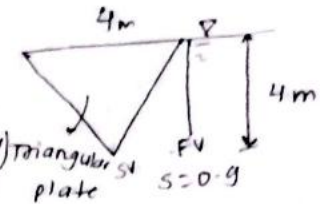
$$= (0.9 \times 9.81) \times \frac{1}{2} \times 4 \times 4 \times \left(\frac{4}{3} \times 4 \right) \text{ Triangular plate}$$

$$= 94.176 \text{ kN}$$

$$y_p = \bar{y} + \frac{I_G}{A \bar{y}}$$

$$= \left(\frac{4}{3} \right) + \frac{\frac{4 \times 4^3}{36}}{\left(\frac{1}{2} \times 4 \times 4 \right) \times \left(\frac{4}{3} \right)}$$

$$= 1.83 \text{ m}$$



④ Solⁿ:

$$F = \gamma A \bar{y}$$

$$= (0.15 \times 9.81) \times \left(\frac{1}{2} \times 2^2 \right) \times 1.5$$

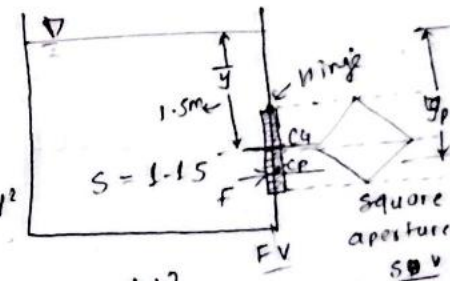
$$= 33.84 \text{ kN}$$

$$A \bar{y} = \frac{1}{2} \times d^2$$

$$y_p = \bar{y} + \frac{I_G}{A \bar{y}}$$

$$= 1.5 + \frac{2 \times \left(\frac{2 \times 1^3}{36} \right)}{\left(\frac{1}{2} \times 2^2 \right) \times 1.5}$$

$$= 1.54 \text{ m}$$



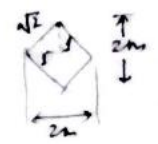
$$A = l^2 = \left(\frac{d}{\sqrt{2}} \right)^2$$

$$l^2 = d^2/2$$

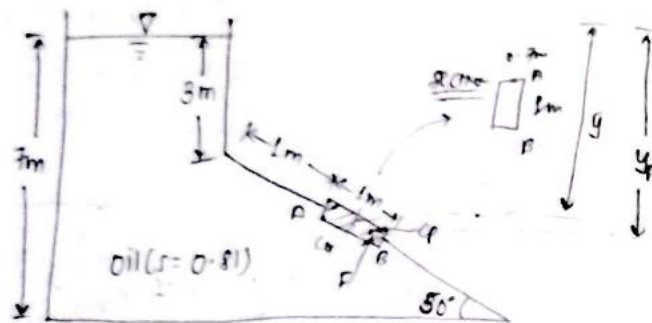
$$[l = 1.41 \text{ m}]$$

$$I_G = \frac{l \times l^3}{36}$$

$$= \frac{l^4}{36} = \frac{(2l^2)^2}{12 \times 4} = \frac{d^4}{48}$$



5



Method I:

Using formula

$$F = \gamma A \bar{y}$$

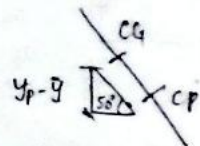
$$= (0.81 \times 9.81) \times (0.7 \times 1) \times \left\{ 3 + \frac{1}{2} \sin 50^\circ \right\}$$

$$= 23.08 \text{ kN}$$

$$y_p = \bar{y} + \frac{I_G \sin^2 \theta}{A \bar{y}}$$

$$= 4.15 + \frac{(0.7 \times 1^3) \sin^2(50^\circ)}{(1 \times 0.7) \times 4.15}$$

$$= 4.162 \text{ m}$$

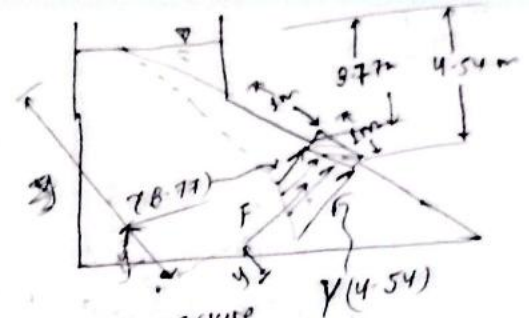


distance b/w cp & cg = $\frac{y_p - \bar{y}}{\sin(50^\circ)} = 0.0157 \text{ m}$

6 Using Pressure dia

$$y_1 = 3 + 1 \sin 50^\circ = 3.77$$

$$y_2 = 3 + 2 \sin 50^\circ = 4.54 \text{ m}$$



F = Volume of pressure dia

$$= \text{Area of trap} \times \text{width}$$

$$= \left[\frac{1(9.77) + 1(14.54)}{2} \right] \times 1 \times 0.7$$

$$= 23.11 \text{ kN}$$

$$y = \frac{1}{3} \left[\frac{2 \times 3.77 + 4.54}{3.77 + 4.54} \right] \quad \left\{ \text{From bottom} \right\}$$

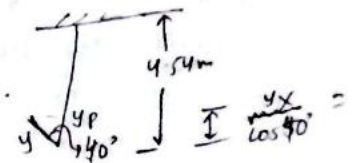
$$= 0.485 \text{ m}$$

$$x = y \cos 40^\circ = 0.371 \text{ m}$$

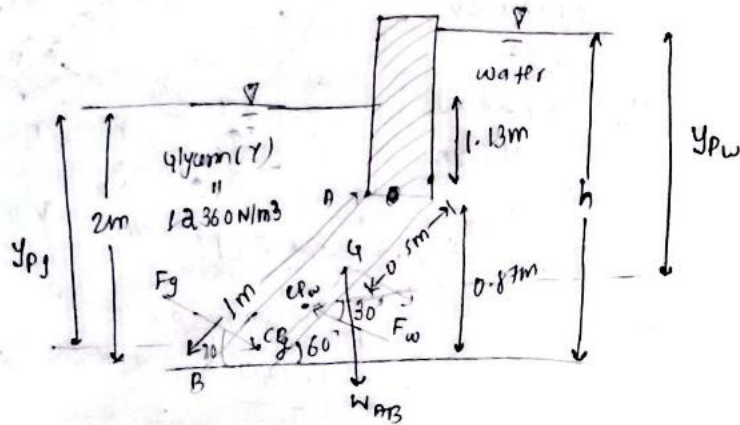
$$y_p = y_2 - x$$

$$= 4.169$$

$$\bar{y} = \frac{y_2 - y_1}{2} + y_1 = 4.15 \text{ m}$$



⑥ Solⁿ



$$(W)_B = 180 \times 9.81 \text{ N}$$

★ चूंकि ABt मा Gate open/close
hunya लकने ABt मा
moment = 0
 $M_B = 0$



$$F_g = \gamma_g A \bar{y}_g$$

$$= 12360 \times (1.2 \times 1) \times \left(2 - \frac{1}{2} \times \sin 60^\circ\right)$$

$$= 23.242 \text{ kN}$$

1.567 m

$$F_w = \gamma_w A \bar{y}_w$$

$$F_w = 9.81 \times 1.2 \times 1 \times (h - 0.5 \sin 60^\circ)$$

$$F_w = (11.772h - 5.097) \text{ kN}$$

$$(y_p)_g = \bar{y}_g + \frac{I_g \sin^2 \theta}{A \bar{y}_g}$$

$$I_g = \frac{1.2 \times 1^3}{12} = 0.1$$

$$= 1.567 + \frac{0.1 \times \sin^2(60^\circ)}{1.2 \times 1.567}$$

$$= 1.607 \text{ m}$$

$$(y_p)_w = \bar{y}_w + \frac{I_g \sin^2 \theta}{A \bar{y}_w}$$

$$= (h - 0.433) + \frac{0.1 \times \sin^2(60^\circ)}{1.2 \times (h - 0.433)} = \frac{(h - 0.433) + 0.0625}{(h - 0.433)}$$

$$(y_p)_w = 1.13 + 0.5 \sin 60^\circ = 1.563 \text{ m}$$

$$M_B = 0$$

$$F_g \times (1.567 - 1.607) - F_w (h - h + 0.433 - \frac{0.0625}{h - 0.433}) + 180 \times 9.81 \times 0.433 = 0$$

$$9.81h - 4.286 = \frac{11.772h - 5.097}{\frac{0.433h - 0.25}{h - 0.433}}$$

$$9.81h - 4.286 = 5.097h^2 - 5.15h + 1.27$$

$$\text{distance b/w AB \& } C_{pg} = 0.5 + C_{pg}$$

$$= 0.5 + \frac{y_{pg} - \bar{y}}{\sin 60^\circ}$$

$$= 0.5 + \frac{(1.607 - 1.563)}{\sin 60^\circ}$$

$$= 0.551 \text{ m}$$

distance b/w A & Cpw = $0.5 + \frac{y_{pw} - (y)_{\text{water side}}}{\sin(60^\circ)}$

$$= 0.5 + \frac{(h - 0.433) + \frac{0.0625}{(h - 0.433)} - (h - \frac{0.87}{2})}{\sin(60^\circ)}$$

$$= 0.5 + \frac{0.0722}{h - 0.433}$$

Thus, For $F_B = 0$

Moment about hinge = 0

$$\sum M_A = 0$$

or, $F_g \times A y_{Pg} - F_w \times A y_{Pw} + W_{AB} \times A g = 0$

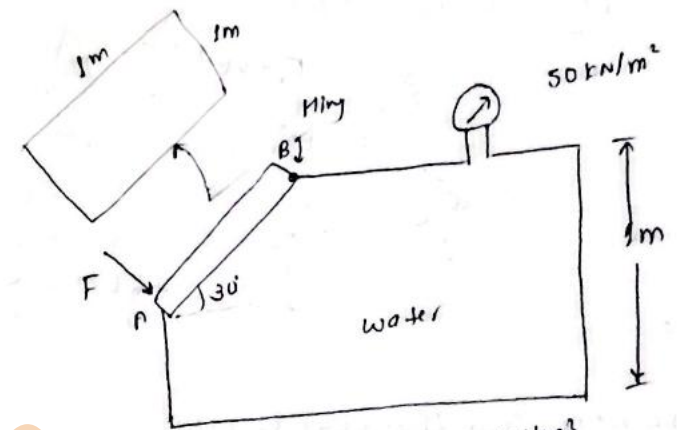
or, $23.242 \times 0.551 - (11.772h - 5.097)(0.5 + \frac{0.0722}{h - 0.433}) + 1.7658 \times 0.5 = 0$

or, $13.689 - 11.772(h - 0.433) \frac{(0.5(h - 0.433) + 0.0722)}{h - 0.433} = 0$

on $13.689 - 5.886h + 2.548 - 0.85 = 0$

$$h = 2.61m$$

(7)



corresponding head for 50 kN/m^2

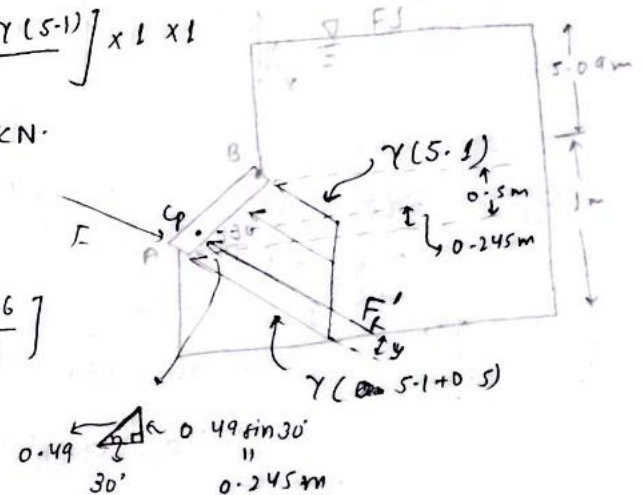
$$= \frac{50 \times 1000}{9.81 \times 1000} = 5.09 \text{ m of water.}$$

Using pressure dia

pressure force

$$F'_R = \left[\frac{\gamma(5.6) + \gamma(5.1)}{2} \right] \times 1 \times 1 = 52.4835 \text{ kN.}$$

$$y = \frac{1}{3} \left[\frac{2 \times 5.1 + 5.6}{5.1 + 5.6} \right] = 0.49 \text{ m}$$



$$\sum M_B = 0$$

$$F \times 1 - F'_R (1 - y) = 0$$

on $\Rightarrow F = 26.77 \text{ kN.}$

Reaction at B = R = ?

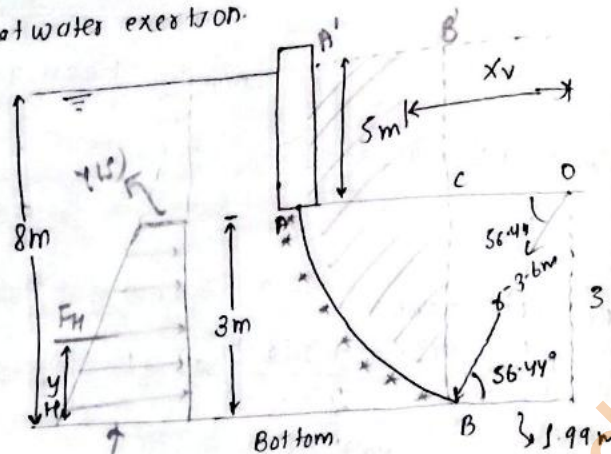
$$\sum F = 0 \quad (\text{K+ve})$$

$$\text{Or, } F - F_R - R = 0$$

$$\begin{matrix} 26.77 & 52.4835 \end{matrix}$$

$$\Rightarrow R = 25.72 \text{ kN}$$

⑧ resultant force that water exertion gate 'AB' per meter length



$$F_H = \frac{\gamma(5) + \gamma(8)}{\gamma(5) + \gamma(8)} \times \frac{1}{3} \left[\frac{\gamma(5) + \gamma(8)}{2} \right] \times 3 \times 1$$

$$\begin{aligned} \text{Type 1: } F_H &= \gamma A \bar{y} \\ &= 9810 \times (3 \times 1) \times \left(5 + \frac{3}{2} \right) \\ &= 191.3 \text{ kN} \quad (\rightarrow) \end{aligned}$$

$$\begin{aligned} \text{Type 2: } F_H &= \text{Volume of pressure diagram} \\ \text{vertical plane} &= \frac{1}{2} \times \frac{1}{2} \left[\gamma(5) + \gamma(8) \right] \times 3 \times 1 \\ &= 191.3 \text{ kN} \quad (\rightarrow) \end{aligned}$$

Location,

From bottom

$$y_H = \frac{1}{3} \left[\frac{2 \times 5 + 8}{5 + 8} \right] \times 3 = 1.385 \text{ m}$$

$$\begin{aligned} (y_p)_{\text{from top}} &= 8 - 1.385 = 6.615 \text{ m} \\ &= 5 + \frac{3}{3} \left[\frac{2 \times 8 + 5}{5 + 8} \right] = 6.615 \text{ m} \end{aligned}$$

Vertical :-

- ① Boundary pt.
- ② wetted part mark
- ③ head zero start
- ④ Area of that part made by curve with zero head
- ⑤ $F_V = \text{Vol}^m \text{ of water content in } A'A B B'A'$

$$\begin{aligned} \frac{AC}{(3.6 - 1.99) \text{ m}} \\ A'A = 5 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Area of sector} &= \frac{360}{360} \rightarrow \pi r^2 \\ 56.44^\circ &\rightarrow \frac{\pi r^2}{360} \times 56.44 \end{aligned}$$

$$\begin{aligned} &= \text{wt} \\ &= \gamma \left[\text{Vol of } A'CB'A' + \text{Vol of } ACBA \right] \\ &= \gamma \left[5 \times (3.6 - 1.99) + \frac{\pi \times 3.6^2 \times 56.44}{360} - \frac{1}{2} \times 2 \times 3 \right] \\ &= 9.81 \left[5 \times 1.6 + 3.38 \right] \\ &= 111.67 \text{ kN} \quad (\uparrow) \end{aligned}$$

$$\begin{aligned} F_R &= \sqrt{F_H^2 + F_V^2} \\ &= 221.51 \text{ kN} \\ \theta &= \tan^{-1} \left(\frac{F_V}{F_H} \right) = 30.27^\circ \quad (\text{with horizontal}) \end{aligned}$$

Area outside of marked area
↓
vertically upward

Location of F_v , From O

Both areas opposite to marked up and down

(v) C.G. of each part

$$\sum M_O = 0$$

$$F_H \times (3 - \frac{y}{2}) - F_v \times (x_v) = 0$$

Resultant horizontal
Resultant vertical

191.3

111.67

$$\Rightarrow x_v = 2.373 \text{ m from 'O'}$$

(9) Horizontal force or wt. of cylinder per meter length. Contact of wall frictionless

curve plane को निम्न Follow करें

तल को Balanced

$$F_H = F_{oil} + F_{water}$$

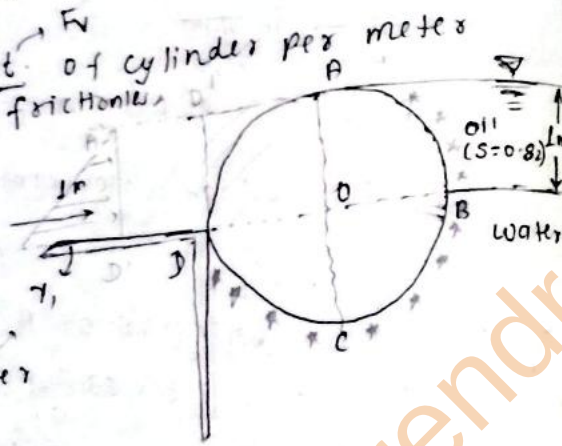
$$F_H = F_{oil} = \gamma A \bar{y}$$

$$= \left(\frac{1}{2} \times \gamma \times 1 \times 1 \right)$$

$$= 40.22 \text{ N}$$

$$= 4.022 \text{ N (←)}$$

$$\begin{aligned} F_H &= \gamma A \bar{y} \\ \text{or } 0.82 \times (1 \times 1) \times (0.5) \times 9810 \\ &= 4.022 \text{ N (←)} \end{aligned}$$



(Fv)

Marked wetted part

Head zero

(Boundary pt) A → A' B → B'

D → D'

D at pressure face

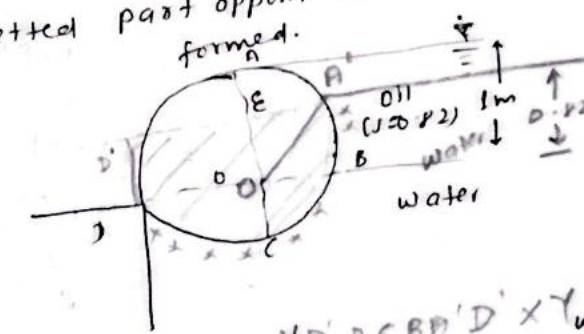
$$\begin{aligned} F_v &= F_{oil} + F_{water} \\ &= \gamma_{oil} V_{ABDD'A} + \gamma_{water} V_{BCDB} \\ &= \gamma_{oil} (V_{AODD'} + V_{ABOA}) + \gamma_{water} V_{BCDB} \\ &= 0.8 \left(1 \times 1 \times 1 + \frac{\pi (1)^2}{4} \times 1 \right) \times 9810 + 9.81 \times \left(\frac{\pi (1)^2}{2} \right) \times 1 \\ &= 29.42 \text{ kN (↑)} \end{aligned}$$

Wetted part opposite to Area formed.

Next method,

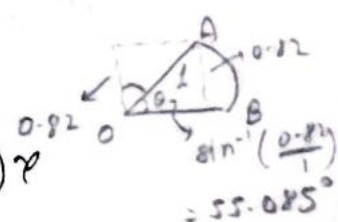
$$\begin{aligned} P_{oil} &= \gamma H = 0.82 \times 9.810 \\ &= 9.81 \times h_w \\ \Rightarrow h_w &= 0.82 \text{ m} \end{aligned}$$

equivalent head (जतासके पानी)

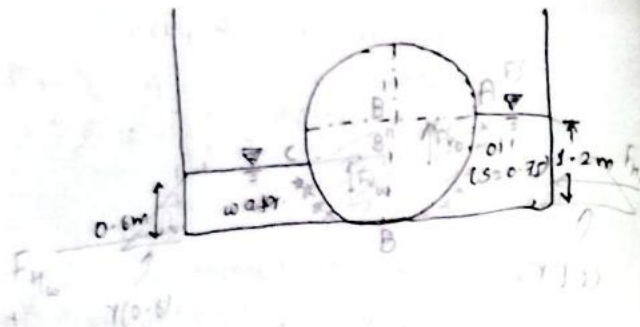


$$F_v = V_{D'DCB A'D'} \times \gamma_w$$

$$\begin{aligned} F_v &= (V_{D'DBE} + V_{EOA'} + V_{A'DB}) \gamma \\ &= \left(0.82 \times 1 \times 1 + \frac{1}{2} \times 0.82 \times (1 \cos 55.085^\circ) \times 1 \right. \\ &\quad \left. + \frac{\pi (1)^2}{360} \times (55.085^\circ + 180^\circ) \right) \times 9.81 \\ &= 30.47 \text{ kN (↑)} \end{aligned}$$



(10)



Prbl 10

$$F_H = F_{H_o} - F_{H_w}$$

$$= \left(\frac{1}{2} \times \gamma_o (1.2) \times 1.2 \right) \times 0.9$$

$$- \left(\frac{1}{2} \times \gamma_w (0.6) \times 0.6 \right) \times 0.9$$

$$= 3.178 \text{ kN} (\leftarrow)$$

$$(F_v)_{oil} = \text{Vol. of } ABB' \times \gamma_{oil}$$

$$= \frac{\pi (1.2)^2}{4} \times 0.9 \times 0.75 \times 9.81$$

$$= 7.849 \text{ kN} (\uparrow)$$

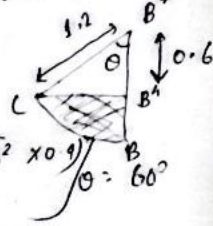
$$(F_v)_{water} = \text{Vol. of } CB''B \times \gamma_w$$

$$= [\text{Vol. of sector } B'CB - \text{Vol. of } \Delta B'CB''] \times \gamma_w$$

$$= \left[\frac{60}{360} \times \pi (1.2)^2 \times 0.9 \right]$$

$$- \frac{1}{2} \times 0.6 \times \sqrt{1.2^2 - 0.6^2} \times 0.9 \times 9.81$$

$$= 3.9 \text{ kN} (\uparrow)$$



Thus

$$\text{Total, } F_v = 7.49 + 3.9$$

$$= 11.39 \text{ kN} (\uparrow)$$

(11)

$$0.2 \text{ kg(f)/cm}^2$$

$$= 0.2 \times 9.81 \times 10^4$$

$$= 19.62 \text{ kN/m}^2$$

$$= \frac{19.62}{9.81} \left(\frac{P}{\gamma} \right)$$

$$0.21 \text{ kg(f)/cm}^2 = 2 \text{ m of water}$$

Boundary pts A & C.

$$F_H = \frac{1}{2} (\gamma(2) + \gamma(3.5)) \times 1.5 \times 2$$

$$= 80.93 \text{ kN} (\rightarrow)$$

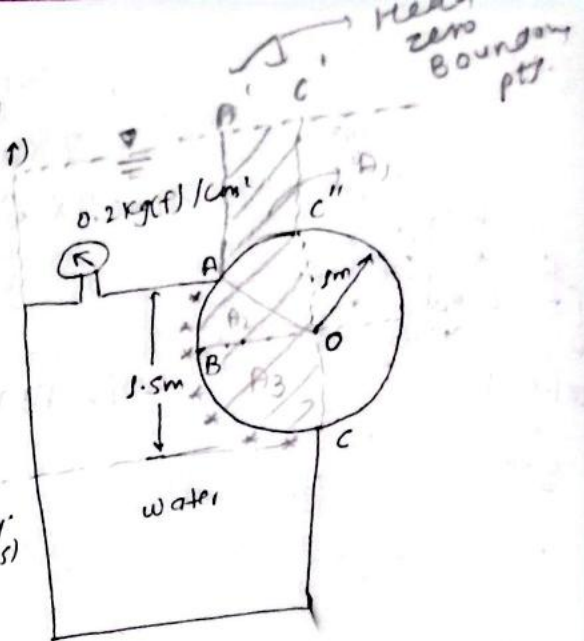
OP'

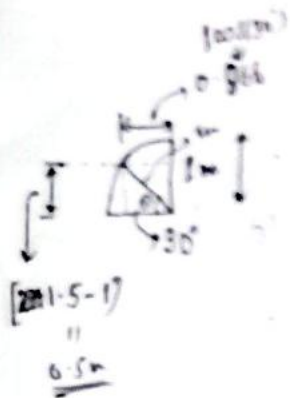
$$F_H = \gamma A \bar{y}$$

$$= 9.81 \times (1.5 \times 2) \times \left(2 + \frac{1.5}{2} \right)$$

$$= 80.93 \text{ kN} (\rightarrow)$$

(F_v) → Boundary pts → head zero
 shaded area
 wt. of shaded portion
 (A'ABCC' acting upward)





$$F_v = \text{Wt. of shaded portion}$$

$$= \gamma \times \text{width} \times \text{Area}$$

$$= \gamma \times 2 \times \left[\frac{A_1}{\gamma} + \frac{A_2}{\gamma} + \frac{A_3}{\gamma} \right]$$

Triangle
Sector
Quarter circle

$$= 9.81 \times 2 \times \left[\frac{1}{2} (2 + 2.5) \times 0.866 \right.$$

$$+ \frac{30}{360} \times \pi (1)^2$$

$$+ \left. \frac{\pi (1)^2}{4} \right]$$

$$= 9.81 \times 2 \times 2.995$$

$$= 58.77 \text{ kN} (\uparrow)$$

↓ off to wetted part

$$A = \int_0^6 y \, dx = \int_0^6 \frac{\pi^2}{4} \, dx = \left[\frac{\pi^2}{4} x \right]_0^6 = 18 \text{ m}^2$$

(12)

$$x_0 = 6 \text{ m}$$

$$y_0 = 9 \text{ m}$$

$$y = \frac{\pi^2}{4}$$

Now

$$F_H = \frac{1}{2} \times \gamma (9) \times 9 \times 1$$

$$= 397.305 \text{ kN} (\rightarrow)$$

$$F_v = \text{Wt. of water in ABOD}$$

$$= \gamma \times \text{width} \times \text{Area of ABOD}$$

$$A_{ABOD} = \int_0^9 x \, dy = \int_0^9 2\sqrt{y} \, dy = \left[\frac{2 \times 2 (y)^{3/2}}{3} \right]_0^9$$

$$= 36 \text{ m}^2$$

$$= 9.81 \times 1 \times 36$$

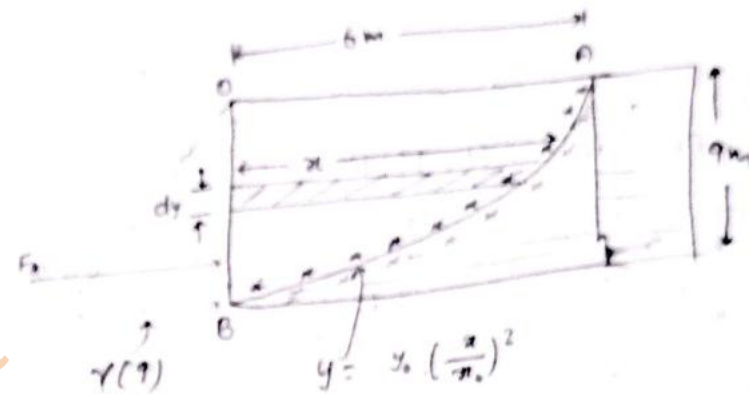
$$= 353.16 \text{ kN} (\downarrow)$$

↓ Towards wetted part

$$\text{Resultant Force } (F_R) = \sqrt{F_H^2 + F_v^2}$$

$$= 531.58 \text{ kN}$$

$$\text{Angle } (\theta) = \tan^{-1} \left(\frac{F_v}{F_H} \right) = 41.63^\circ$$



13

Pressure = -117.72 kPa

or, $\gamma h_{\text{liquid}} = -117.72$

or, $h_{\text{liquid}} = \frac{-117.72}{1.2 \times 9.81}$

$h_{\text{liquid}} = -10\text{m}$

So, making head zero

$F_H = \text{Wt of pressure dia}$

$= \left\{ \frac{1}{2} \times (\gamma) 8.2 \times 8.2 \right\} \times 10 \times 1.2 \times 9.81$

$= 3957.75 \text{ kN} (\rightarrow)$

or, $F_H = \gamma A \bar{y}$

$= (1.2 \times 9.81) \times (8.2 \times 10) \times \left(\frac{8.2}{2} \right)$

$= 3957.75 \text{ kN} (\rightarrow)$

$(y_p)_H = \frac{2}{3} \times 8.2 = 5.47 \text{ m from FS}$

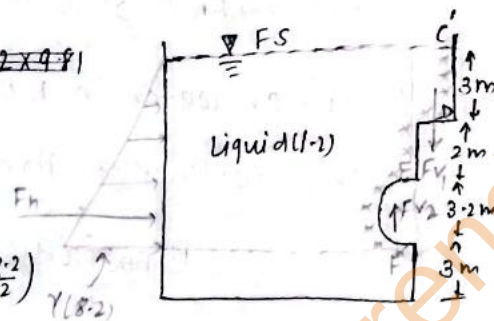
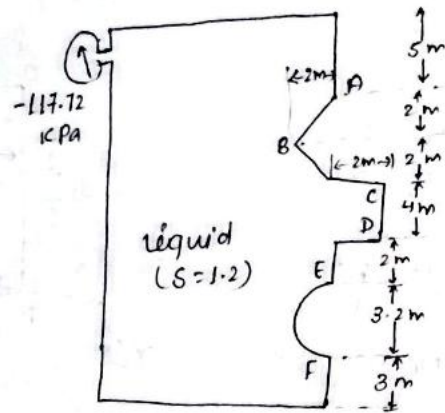
Now,

$F_V = \text{wt. of liquid in curved portion upto F.}$

$= F_{V1} - F_{V2}$

$= [3 \times 2 \times 10 \times 1.2 \times 9.81 - \frac{\pi (1.6)^2}{2} \times 10 \times 1.2 \times 9.81]$

$= 232.94 \text{ kN} (\downarrow)$



$F_{V1} \text{ acts } \rightarrow \frac{2}{2} = 1\text{m left of CD}$

$F_{V2} \text{ acts } \rightarrow \left(2 + \frac{4 \times 1.6}{3\pi} \right) = 2.68 \text{ m from CD}$

Thus,

Varignon's Theorem abt. CD,

$F_{V1} \times 1 - F_{V2} \times 2.68 = F_V \times x_V$

or, $\frac{3 \times 2 \times 10 \times 1.2 \times 9.81}{2} - \frac{\pi \times (1.6)^2}{2} \times 10 \times 1.2 \times 9.81 = 473.38 \times x_V$

$x_V = 2.41 \text{ from CD}$

$\frac{3 \times 2 \times 10 \times 1.2 \times 9.81}{2} = 706.32 \text{ N}$

Buoyancy & Flootation

①

Wt of body

$$= (60 \times 0.4) \times \gamma_w$$

$$+ (Vol \text{ of } V) \times \gamma_{\text{ing}}$$

$$15.6 \times 9.81$$

$$V \times \gamma_w$$

$$5 \times 9.81$$

$$5 = 6.04 \times 10^3 \text{ kg/m}^3$$

concrete
v = ?
(s = 2.5)



②

(a) Block completely immersed in water

$$(4 \times 10.5) \text{ m}^3$$

$$s = 0.76$$

$$\text{Wt of Block \& concrete} = (\text{Vol}^e \text{ of Block}) \times \gamma_w$$

$$(4 \times 10.5 \times 0.76 + V_c \times 2.5) = (4 \times 10.5) \times 1$$

$$V_c = 0.192 \text{ m}^3$$

(b) Block \& concrete completely immersed in water.

$$\text{Wt of Block \& concrete} = (\text{Vol}^e \text{ of Block \& concrete}) \times \gamma_w$$

$$(4 \times 10.5 \times 0.76 + V_c \times 2.5) = (4 \times 10.5 + V_c) \times 1$$

$$V_c = 0.32 \text{ m}^3$$

③

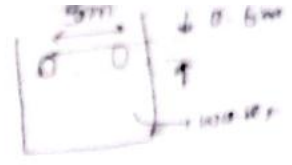
$$\text{Wt of log} = (\text{Vol}^e \text{ of log}) \times \gamma_w$$

$$\text{or } \left(\frac{\pi \times 0.6^2}{4} \times 5 \times 0.7 \right)$$

$$= (V_{\text{log}})_{\text{immersed}} \times 1$$

$$(V_{\text{log}})_{\text{immersed}} = 0.4896 \text{ m}^3$$

depth in water =



$$S_{\text{log}} = 0.7$$

$$A_{\text{immersed}} \times \text{length} = V_{\text{immersed}}$$

$$\left(\frac{\pi d^2}{4} \times \frac{\theta}{360} - \frac{1}{2} s^2 \sin \theta \right) \times 5 = 0.4896$$

Solving,

$$\theta = 217.288$$

$$\cos \left(\frac{\theta}{2} \right) = \frac{s-h}{s}$$

$$\Rightarrow h = 0.397 \text{ m}$$

$$A_{\text{immersed}} = A_{\text{sector}} - A_{\Delta}$$

$$= \frac{\pi d^2}{4} \times \frac{\theta}{360}$$

$$- \frac{1}{2} s^2 \sin \theta$$

$$\frac{1}{2} s^2 \sin \theta$$

Area of triangle

of equal side 's'

subtending angle θ

from vertex

from vertex

from vertex

④

$$265 + W = 0.18 \times 1 \times 9.81 \times 1000$$

$$V \times 85 \times 9.81 \times 1000$$

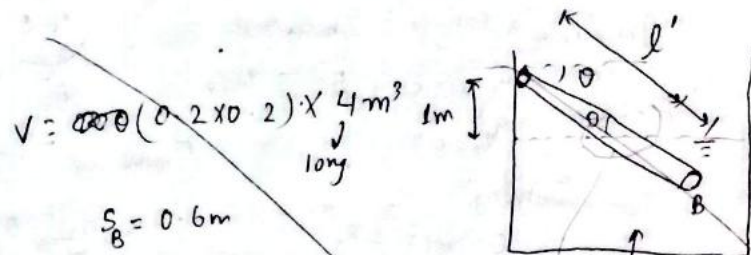
$$\Rightarrow S_B = 0.85$$

If, $F = 157N$

$$157 + 0.18 \times 0.85 \times 9.81 \times 1000 = 0.18 \times S_L \times 9.81 \times 1000$$

$$\Rightarrow S_L = 0.939$$

⑤



$$S_B = 0.6m$$

$$Wt. \text{ of Beam} = (Vol. \text{ immersed}) \times \gamma_w$$

$$\text{or, } 0.2 \times 0.2 \times 4 \times 0.6$$

$$= [0.2 \times 0.2 \times (4 - \frac{1}{\sin \theta})$$

$$+ \frac{1}{2} \times \frac{0.2}{\tan \theta} \times 0.2 \times 0.2]$$

$$\times 1$$

$$h = \frac{0.2}{\sin \theta}$$

$$L' = \frac{1}{\sin \theta}$$

$$L_{im} = 4 - \frac{1}{\sin \theta}$$

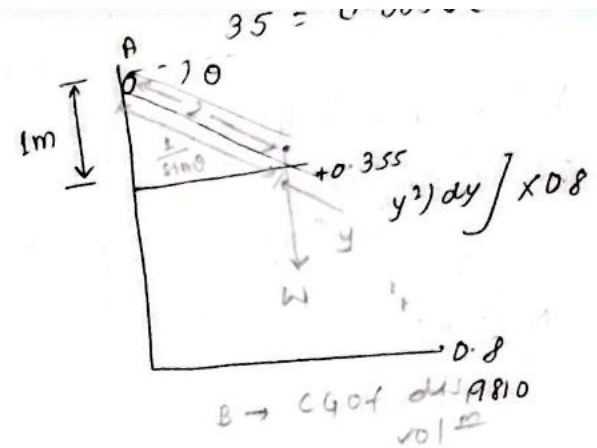
⑥

$$AB = 4m.$$

$$C/S = 0.2 \times 0.2 m^2$$

$$y = 4 - \frac{1}{\sin \theta}$$

⚡ (A) → hinged
⚡
 $\Sigma M_A = 0$



$$\Sigma M_A = 0$$

$$W \times AG' - F_B \times AB' = 0$$

$$W = \frac{(0.2 \times 0.2 \times 4) \times 0.6 \times 9810}{Vol}$$

$$= 941.76 N$$

$$F_B = \text{wt. of water displaced}$$

$$= \gamma \times V \text{ displaced}$$

$$= 9810 \times (0.2 \times 0.2 \times y)$$

$$= 392.4 y$$

$$AG' = 2 \cos \theta$$

$$AB' = (2 + \frac{y}{2}) \cos \theta = (4 - \frac{y}{2}) \cos \theta$$

Thus,

$$941.76 \times 2 \cos \theta - 392.4 y (4 - \frac{y}{2}) \cos \theta = 0$$

or, solving

$$y = \frac{6.53 m}{> 4m \times} \quad \& \quad y = 1.47m$$

⚡ $W \neq F_B$ since,
hinge पर
some rxn लागे
(होए)

$$\rightarrow y = 4 - \frac{1}{\sin \theta}$$

$$\Rightarrow \theta = 23.3^\circ$$

④

$$2650 + W = 0.18 \times 1 \times 9.81 \times 1000$$

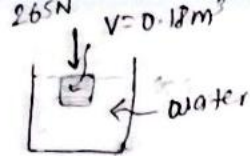
$$V \times \frac{85}{100} \times 9.81 \times 1000$$

$$\Rightarrow S_B = 0.85$$

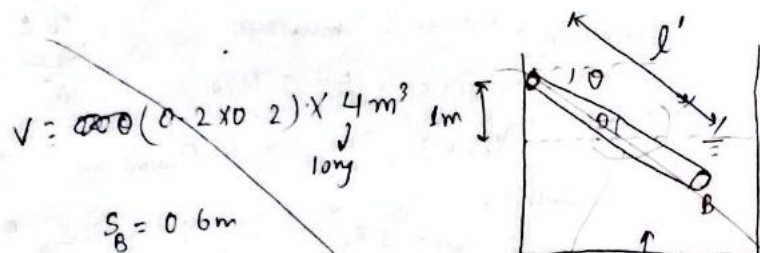
$$\text{If, } F = 157 \text{ N}$$

$$157 + 0.18 \times 0.85 \times 9.81 \times 1000 = 0.18 \times S_L \times 9.81 \times 1000$$

$$\Rightarrow S_L = 0.939$$



⑤



$$\text{Wt. of Beam} = (\text{Vol. immersed}) \times \gamma_w$$

$$\text{or, } 0.2 \times 0.2 \times 4 \times 0.6$$

$$= [0.2 \times 0.2 \times (4 - \frac{1}{\sin \theta})$$

$$+ \frac{1}{2} \times \frac{0.2}{\tan \theta} \times 0.2 \times 0.2]$$

$$\times 1$$

$$h = \frac{0.2}{\sin \theta}$$

$$L' = \frac{1}{\sin \theta}$$

$$\text{lim} = 4 - \frac{1}{\sin \theta}$$

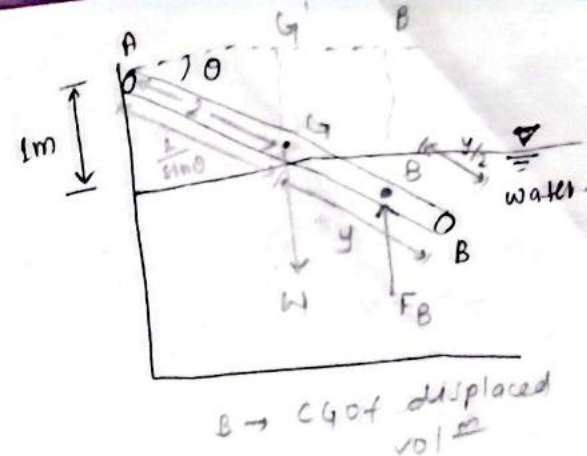
⑥

$$AB = 4 \text{ m}$$

$$C/S = 0.2 \times 0.2 \text{ m}^2$$

$$y = 4 - \frac{1}{\sin \theta}$$

$$\star \text{ (A) } \rightarrow \text{ hinged } \downarrow \sum M_A = 0$$



$$\sum M_A = 0$$

$$W \times AG' - F_B \times AB' = 0$$

$$W = \frac{(0.2 \times 0.2 \times 4) \times 0.6 \times 9810}{\text{Vol}} \quad W = \gamma$$

$$= 941.76 \text{ N}$$

$$F_B = \text{wt. of water displaced}$$

$$= \gamma \times V_{\text{displaced}}$$

$$= 9810 \times (0.2 \times 0.2 \times y)$$

$$= 392.4 y$$

$$AG' = 2 \cos \theta$$

$$AB' = (2 + \frac{y}{2}) \cos \theta = (4 - \frac{y}{2}) \cos \theta$$

Thus,

$$941.76 \times 2 \cos \theta - 392.4 y (4 - \frac{y}{2}) \cos \theta = 0$$

or, solving

$$y = 6.53 \text{ m} \quad \& \quad y = 1.47 \text{ m}$$

$W \neq F_B$ since, hinge पर some rxn लागेको हुनेछे)

$$\rightarrow y = 4 - \frac{1}{\sin \theta} \Rightarrow \theta = 23.3^\circ$$

⑥

dia of float (d) = 15cm

$$\sum M_{\text{about Hinge}} = 0$$

$$\sum M_O = 0$$

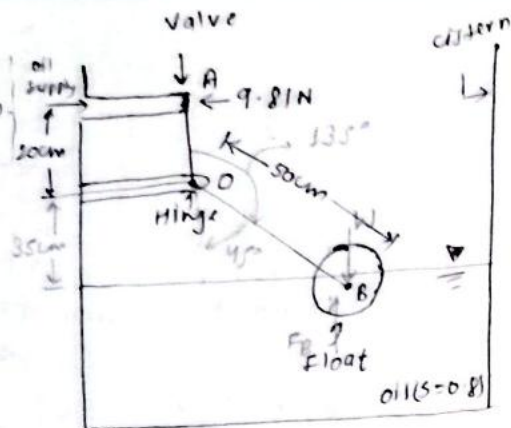
So,

$$F_B \times 50 \sin(45^\circ) - W \times 50 \cos(45^\circ)$$

$$+ 9.81 \times 20 = 0$$

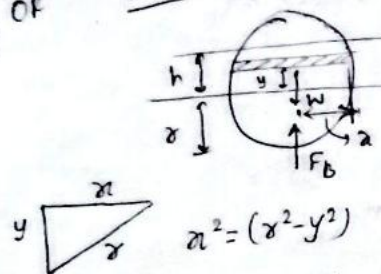
$$W = \text{Vol.} \times \gamma_{\text{float}}$$

$$F_B = (\text{Vol.})_{\text{immersed}} \times \gamma_{\text{oil}}$$



(W & F_B passes thr. same line being symmetrical float)

③ 'OF' Analytical



$$dV = \pi x^2 dy$$

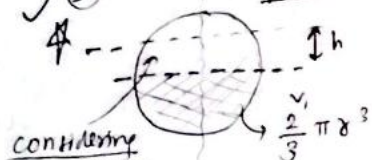
$$dV = \pi (r^2 - y^2) dy$$

(From center to ht. h)

$$V = \int_0^h \pi (r^2 - y^2) dy$$

$$V = \frac{\pi}{3} [(h+r)^2 (3r - (h+r))] - \frac{\pi}{3} [3r^2 h + 2r^3 - h^3]$$

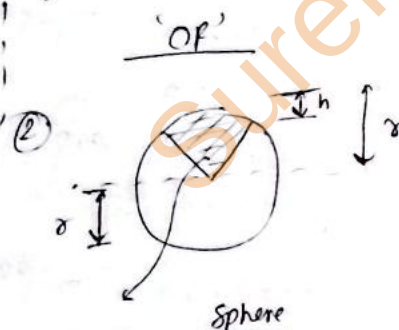
can neglect h if very small.



considering cylinder

$$V_1 = \pi r^2 \times h$$

approx.



$$h = 50 \sin 45^\circ - 35 = 0.007157 \text{ m}$$

Thus,

$$F_B = (\text{Vol.})_{\text{immersed}} \times \gamma_{\text{oil}}$$

$$= \left[\frac{2}{3} \pi \left(\frac{0.15}{2} \right)^3 + \int_{0.075}^{0.355} \pi (r^2 - y^2) dy \right] \times 0.8$$

'OF'

$$\left[\frac{2}{3} \pi \left(\frac{0.15}{2} \right)^3 + \pi \left(\frac{0.15}{2} \right)^2 \times 0.355 \right] \times 0.8 \times 9810$$

$$= 70.21 \text{ N} \quad 74.27 \text{ N}$$

Thus,

$$F_B \times 35.35 - W \times 35.35 + 196.2 = 0$$

$$\Rightarrow W = 1.877 \text{ N}$$

$$F_B \times 35.35 + 9.81 \times 20 - W \times 35.35 = 9.81 \times 20$$

(Moment left side & right side of hinge must be equal)

$$\{ \sum M_{\text{left}} = \sum M_{\text{right}} \}$$

7

Vol^e displaced by curve = 0.6 m³

Wt. of displaced liquid = 3.9 tonnes

$$= 3.9 \times 9810 \text{ N}$$

$$= 38259 \text{ N}$$

Metacentric Ht (GM) = ?

Say, metacenter M.

$$GM = BM - BG = \frac{I}{V} - BG \quad \text{--- (1)}$$

$$I = \frac{\pi D^4}{64}$$

$$= \frac{\pi \times 3^4}{64}$$

$$= 3.976 \text{ m}^4$$

✓ (FS ले काटेको surface
हने: Here circular)

★ (Surface ले काटेको position
को I निकालने) plan

$$\therefore \text{displaced vol}^e, V = \frac{3.9 \times 9810}{9810} = 3.9 \text{ m}^3$$

★ BG → extra end बाट B सम्म — G सम्मको
(तल वा माथि) distance distance

$$BG = OB - OG$$

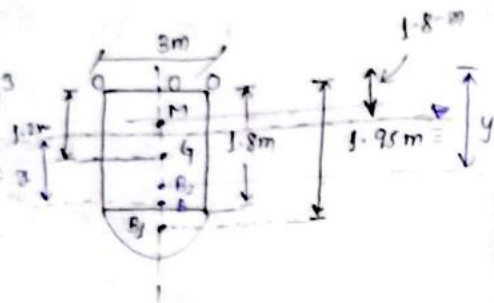
$$= OB - 1.2$$

✓ Use Archimedes principle,

$$W_{\text{Body}} = W_{\text{vol displaced (Cylinder + curve)}}$$

$$\text{or, } 3.9 \times 9810 = 9810 \left[\underbrace{0.6}_{\text{vol displaced by curve}} + \underbrace{\frac{\pi D^2}{4} \times x}_{\text{by cylinder}} \right]$$

$$x = 0.47 \text{ m}$$



$$OB_1 = 1.95 \text{ m}$$

$$OB_2 = (1.8 - x) + \frac{x}{2}$$

submerged portion को centroid

$$OB_2 = 1.566 \text{ m}$$

curved part

cylindrical portion

$$OB \times V_{\text{dup (total)}} = OB_1 \times V_1 + OB_2 \times V_2$$

$$OB \times \frac{3.9 \times 9810}{9810} = 1.95 \times 0.6 + 1.566 \times \frac{\pi d^2}{4} \times 0.466$$

$$\begin{aligned} \text{Overall CG,} \\ OG \times W_T &= \\ OG_1 \times W_1 &+ \\ OG_2 \times W_2 &+ \end{aligned}$$

$$OB = 1.629 \text{ m}$$

$$BG = OB - 1.2 = 0.429 \text{ m}$$

$$GM = \frac{3.976}{3.9} - 0.429 = 0.597 \text{ m}$$

8

State, if it can flow vertically in water,

$$GM \geq 0 \text{ (stable)}$$

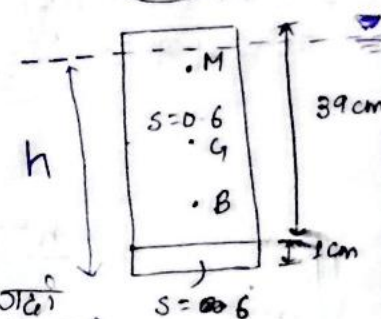
$$GM = BM - BG$$

$$= \frac{I}{V} - BG$$

$$I = \frac{\pi d^4}{64} \text{ (cylindrical portion cut + } \frac{\pi d^4}{64} \text{ circular section } \frac{\pi d^4}{64} \text{)}$$

$$= 4.91 \times 10^{-6} \text{ m}^4$$

10 cm



S = 0.6

S = 0.6

S = 0.6

S = 0.6

S = 0.6

S = 0.6

S = 0.6

S = 0.6

S = 0.6

S = 0.6

S = 0.6

$$V = \frac{\pi D^2}{4} \times h$$

$$B_G = O_G - O_B$$

can't do $\frac{40}{2}$ } → diff material

$$O_G \times W_{\text{total}} = O_{G_1} W_1 + O_{G_2} W_2$$

(W₁ + W₂)

$$\text{or } O_G \times \frac{9810 \times \pi \times 10^2}{4} (0.6 \times 39 + 6 \times 1) = O_{G_1} \times \frac{0.6 \times 9810 \times \pi \times 10^2}{4} \times 39 + \frac{39}{2} + \frac{1}{2} (39 + \frac{1}{2}) (6 \times 9810 \times \frac{\pi \times 10^2}{4} \times 1)$$

$$O_G = 23.58 \text{ cm.}$$

OB = ?

✓ Archimedes principle,

$$W_{\text{body}} = W_{\text{liquid displaced}}$$

(γ₁V₁ + γ₂V₂) (γ_w × V)

$$\text{or, } 9810 \times \frac{\pi \times 10^2}{4} (0.6 \times 39 + 6 \times 1) = \frac{\pi \times 10^2}{4} \times 9810 \times 1 \times h$$

$$h = 29.4 \text{ cm} \} \rightarrow \text{depth inside water}$$

$$OB = (40 - 29.4) + \frac{29.4}{2} = 25.3$$

$$\begin{aligned} B_G &= O_G - O_B \\ &= 23.58 - 25.3 \\ \text{length} &= 1.72 \text{ cm} \end{aligned}$$

Thur,

$$GM = \frac{I}{V} - BG$$

$$= \frac{4.91 \times 10^{-6}}{\frac{\pi \times (0.1)^2}{4} \times \frac{1.72 \times 10^{-2}}{29.4 \times 10^{-2}}} - (1.72 \times 10^{-2})$$

$$= -0.01507 \text{ m}$$

= -1.507 cm, so, it can't float in water

⑨ Solⁿ:-

long → 10m

$$W_{\text{piston}} = 686.7 \text{ kN}$$

d_{upper deck} = 5m
(boiler)

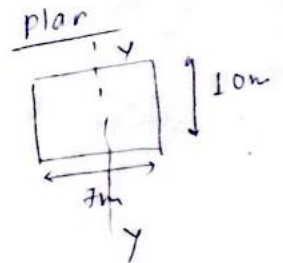
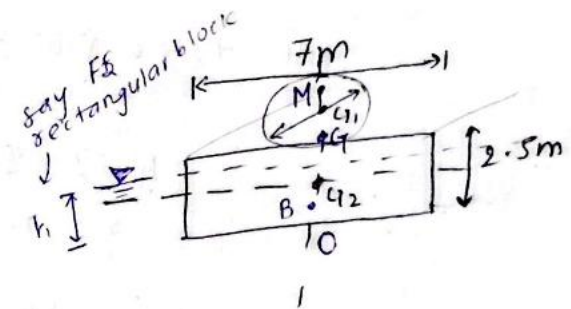
$$W_{\text{deck (boiler)}} = 588.6 \text{ kN}$$

$$\rho_w = 10.04 \text{ kN/m}^3$$

$$\text{Metacentric Ht, } GM = \frac{I}{V} - BG$$

- overall G
- overall B

$$I = \frac{1}{12} \times 10 \times 7^3 = 285.83 \text{ m}^4$$



$$OG \times W = OG_1 \times W_1 + OG_2 \times W_2$$

$$OG (686.7 + 588.6) = (2.5) \times 588.6 + (2.5) \times 686.7$$

$$OG = 2.98 \text{ m from O}$$

Assume FF

OB → Archimedes → $\left(\frac{h}{2}\right)$

$W_{\text{Body}} = W_{\text{displaced}}$

$$(686.7 + 588.6) \times 10^3 = \frac{(10 \times 7 \times h) \times 10.04 \times 10^3}{\gamma_w}$$

$h = 1.815 \text{ m}$

$OB = \frac{h}{2} = 0.908 \text{ m}$

$BG = OG - OB = 2.073$

Metacentric ht $GM = \frac{I}{V} - BG$

$$= \frac{285.83}{(10 \times 7 \times 1.81)} - 2.073$$

$= 0.183 \text{ m}$

Q10 Solⁿ:

$OB = h/2$

$OG = L/2$

$GM \geq \frac{I}{V} - BG$

Archimedes

$W_{\text{Body}} = W_{\text{displaced}}$

$$\frac{\pi d^2}{4} \times L \times 0.6 = \frac{\pi d^2}{4} \times h \times 0.8$$

$\Rightarrow h = 0.75L$ — (1)

$I = \frac{\pi D^4}{64}$

$V = \frac{\pi D^2}{4} \times h$

$OG = \frac{L}{2}$; $OB = \frac{h}{2} = \frac{0.75L}{2}$

$GM \geq \frac{\pi D^4}{64 \times \pi D^2 \times h} \times 4 - 0.125L$

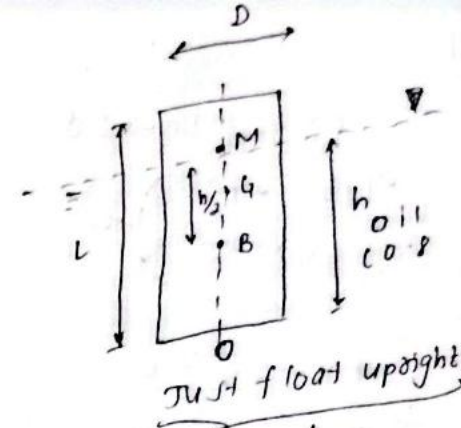
$GM = \frac{D^2}{12L} \geq 0.125L \geq 0$

↓

0

$D^2 \geq 1.5L^2$

$\frac{L}{D} = \sqrt{\frac{1}{1.5}} = 0.816$



$GM \geq 0$

$Lh \rightarrow$ Archimedes

$BG = OG - OB = 0.125L$

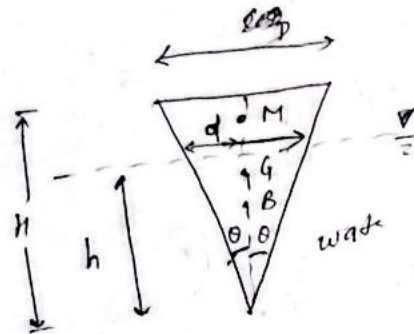
(11)

Apex angle $\rightarrow 2\theta$

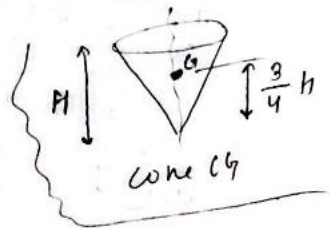
Stable eq^m
 $GM \geq 0$

$$BM - BG \geq 0$$

$$\frac{I}{V} - BG \geq 0$$



* I $\rightarrow$ $\int r^2 dm$ part को
 plan को MOI
 about longitudinal
 axis.



$$\left(\frac{\pi d^4}{64} \right) - (OG - OB) \geq 0$$

Vol^m of submerged portion of cone

Archimedes

 $W_{\text{body}} = W_{\text{displaced}}$

$$\text{or, } \left(\frac{\pi D^2}{4} \times \frac{H}{3} \right) \times 0.7 \times 9810$$

$$= \left(\frac{\pi d^2}{4} \times \frac{h}{3} \right) \times 1 \times 9810$$

$$h = 0.7 \frac{D^2 H}{d^2}$$

$$= 0.7 \frac{4H^2 \tan^2 \theta H}{d^2} = \frac{0.7 H^3}{h^2}$$

$$h = \frac{2.8 H^3 \tan^2 \theta}{d^2} \quad \left(\because \frac{0.7 H^3}{h^2} \right)$$

$$h^3 = 0.7 H^3$$

$$\sqrt[3]{h} = 0.8879 H$$

$$H = 1.126 h$$

$$\text{or, } \frac{3d^2}{16 \times h} - \frac{3}{4} (H - h) \geq 0$$

$$\text{or, } \frac{9 \times d^2}{16 \times h} - \frac{3}{4} (1.126h - h) \geq 0$$

$$\text{or, } \frac{3d^2}{16h} - 0.0945h \geq 0$$

$$\text{or, } 3d^2 \geq 1.512h^2$$

$$\text{or, } \frac{3}{1.512} \geq \frac{h^2}{d^2}$$

$$\text{or, } \frac{3}{1.512} \geq \frac{1}{4 \tan^2 \theta}$$

$$\text{or, } \tan^2 \theta \geq 0.126$$

$$\Rightarrow \theta \geq 19.54^\circ$$

Thus least apex angle = 2θ
 $= 2 \times 19.54^\circ$
 $= 39.08^\circ$

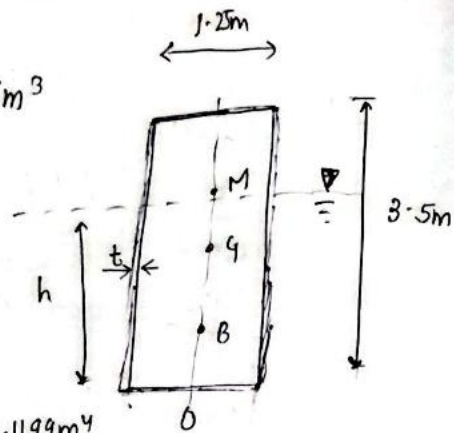
(12)

$$w_{\text{cylinder}} = 75.537 \text{ kN/m}^3$$

Stable eqⁿ.

$$GM \geq 0$$

$$\frac{I}{V} - BG \geq 0$$



$$\textcircled{1} \quad I = \frac{\pi D^4}{64} = \frac{\pi (1.25)^4}{64} = 0.1199 \text{ m}^4$$

$$\gamma_w = w_{\text{water}} = 10.055 \text{ kN/m}^3$$

$t \rightarrow$ considered small
So, $t^2 \rightarrow$ neglected

Archimedes,

$$w_B = W_{\text{displaced water}}$$

$$\text{Or, } \frac{(75.537 \times 10^3) \times V_B}{\gamma_B} = \frac{(10.055 \times 10^3)}{\gamma_w} \times \left(\frac{\pi \times D^2}{4} \right) \times h \quad \text{--- (1)}$$

$$\textcircled{2} \quad V_B = \left[\frac{\pi D^2 \times 3.5}{4} - \frac{\pi (D-2t)^2}{4} \right] (3.5-2t)$$

$$h = 6.122 [1.227 - 0.785(1.25-2t)^2] \times 3.5$$

$$h = 21.427 [1.227 - 0.785(1.25-2t)^2] \quad \text{--- (1)}$$

$$\text{Thus, } \frac{I}{V} - BG \geq 0$$

$$BG = 0.08 \\ = 1.75 - \frac{h}{2}$$

$$\frac{0.1199}{\frac{\pi D^2 \times h}{4}} - (1.75 - \frac{h}{2}) \geq 0$$

$$\text{Or, } \frac{0.0977}{h} - \left(\frac{3.44-h}{2} \right) \geq 0$$

$$\text{Or, } 0.1954 - 3.44h + h^2 \geq 0$$

Solving

$$h = 3.3822 \text{ m}$$

$$\text{Or, } h = 0.0577 \text{ m}$$

$$\text{For } h = 3.3822 \text{ m,}$$

from (1),

$$t = 1.208 \text{ m}$$

$$V_B = 4.295 = \frac{\pi}{4} (1.25^2 - 2 \times 1.25 \times 2t + 4t^2) (3.5-2t)$$

$$V_B = 4.295 = \frac{\pi}{4} (1.5625 - 5t) (3.5-2t) \quad \text{from (1)}$$

$$V_B = 4.295 = 4.295 + 16.198t$$

$$\text{Thus, } \frac{I}{V} - BG \geq 0$$

$$\frac{0.1199}{\frac{\pi \times 1.25^2}{4} \times 99.163t} - (1.75 - 49.58t) \geq 0$$

$$\text{Or, } \frac{0.000985}{t} - 1.75 + 49.58t \geq 0$$

$$\text{Or, } 0.000985 - 1.75t + 49.58t^2 \geq 0$$

$$\Rightarrow t = \frac{0.000985}{1.75} = 0.563 \text{ mm}$$

(13)

$W_{\text{Body}} = W_{\text{vol}} = \text{displaced}$
 or $(450 + 4.5 \times 1000) = \left(\frac{\pi \times 1.5^2}{4} \times x \right) \times 1.025 \times 9810$

$$x = 0.28 \text{ m}$$

(13)

$$W_B = 4.5 \text{ kN} = 4500 \text{ N}$$

$$W_A = 450 \text{ N}$$

$$S_{sw} = 1.025$$

From,
Archimedes principle,

$$W_{\text{Body}} = W_{\text{vol}} = \text{displaced}$$

$$\text{or, } W_A + W_B = V \times \gamma_{sw}$$

$$\text{or, } V = \frac{(4500 + 450)}{1.025 \times 9810} = 0.4923 \text{ m}^3$$

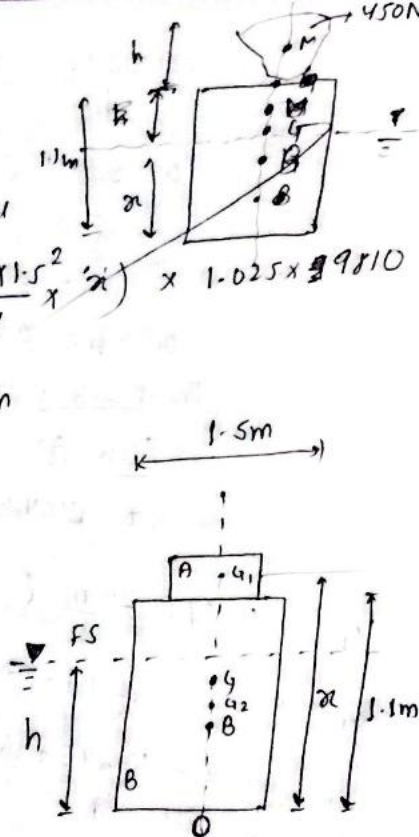
$$h = \frac{V}{A_{cs}} = \frac{0.4923}{\frac{\pi \times 1.5^2}{4}} = \frac{0.4923}{1.767} = 0.278 \text{ m}$$

For combined G we have,

$$W_T \times OG = W_1 \times OG_1 + W_2 \times OG_2$$

$$\text{or, } 4950 \times OG = 450 \times x + 4050 \times 0.55$$

$$\text{or, } OG = \bar{x} = 0.091x + 0.5 - \text{①}$$



$$I = \frac{\pi D^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.248 \text{ m}^4$$

$$V = 0.4923 \text{ m}^3$$

$$\frac{I}{V} = 0.5048 \text{ m}$$

For eq^m

$$GM \geq 0$$

$$BM - BG \geq 0$$

$$\text{or, } \frac{I}{V} - (OG - OB) \geq 0$$

$$\text{or, } 0.5048 - 0.091x - 0.361 \geq 0$$

$$0.1438 \geq 0.091x$$

$$\text{or, } x \leq 1.58 \text{ m}$$

$$\text{Max } x = 1.58 \text{ m}$$

So, CG of load is 1.58 m above bottom of cylindrical buoy & (1.58 - 1.1 = 0.48 m) from top of buoy.

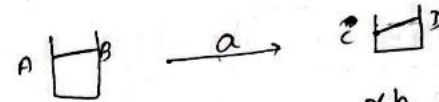
$$BG = OG - OB$$

$$= (0.091x + 0.5) - \frac{h}{2}$$

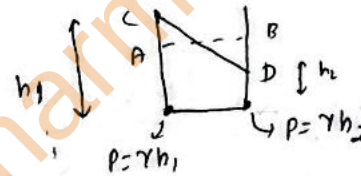
$$BG = 0.091x + 0.361$$

Liquids in Relative Equilibrium

- Liquid & its container in motion but relative to container, liquid is at equilibrium.
- Liquid level changes ~~however~~ relative.



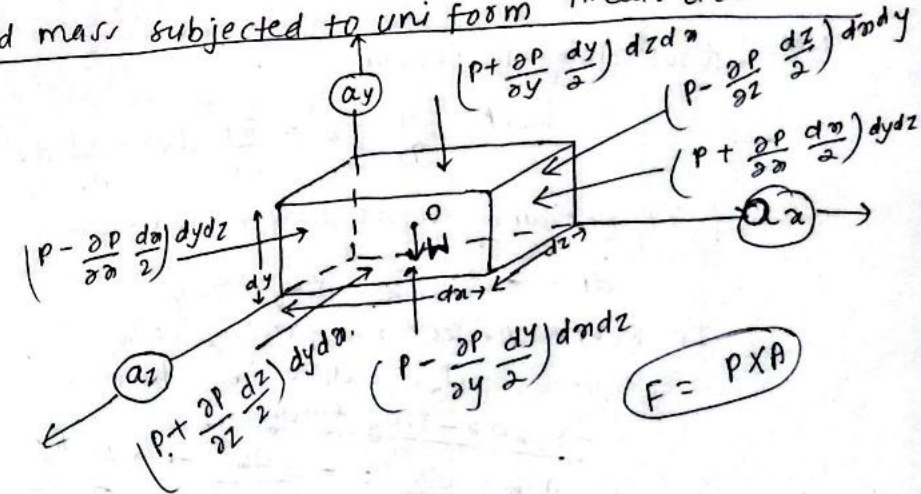
However, we use, $P = \gamma h$ for pressure determination
(i.e. Hydrostatic law is used
pressure increases with depth)



- Pressure acts normally.

- Acceleration/Motion can be any type horizontal, vertical, inclined, rotational.

Fluid mass subjected to uniform linear acceleration



Newton's law

$$\sum F_x = m a_x \quad (- \rightarrow +ve)$$

$$\text{or } - \frac{\partial P}{\partial x} (dx dy dz) = (\rho dx dy dz) a_x$$

$$\boxed{\frac{\partial P}{\partial x} = -\rho a_x}$$

$$\sum F_z = m a_z$$

$$\boxed{\frac{\partial P}{\partial z} = -\rho a_z}$$

$$\sum F_y = 0$$

$$\sum F_y = m a_y$$

$$- \frac{\partial P}{\partial y} (dx dy dz) = m a_y$$

$$\boxed{\frac{\partial P}{\partial y} = -\rho(a_y + g)}$$

$$W = mg$$

↓
SV g

Note: total change in pressure,

$$dP = \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy + \frac{\partial P}{\partial z} dz$$

If pressure varies only in x & y dir.

$$dP = -\rho a_x dx - \rho(a_y + g) dy$$

If surface of constant pressure, $dP = 0$

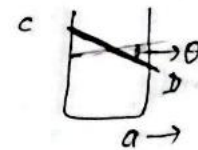
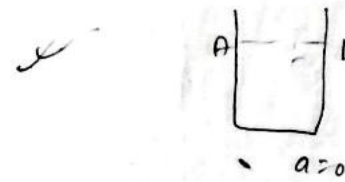
$$P_2 - P_1 = dP = 0 \quad \text{b/w two pts}$$

$$-\rho a_x dx - \rho(a_y + g) dy = 0$$

$$\therefore \text{slope} = \frac{dy}{dx} = - \frac{a_x}{(g + a_y)} \quad \text{of liq surface.}$$

$$m = \rho V$$

$$= \rho dx dy dz$$

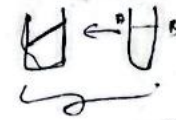
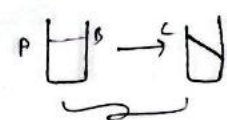
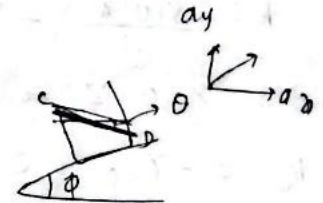
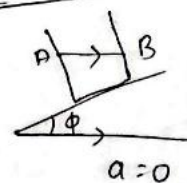


$\frac{dy}{dx} \rightarrow$ FS/CD
को slope
↓
CD गति pressure
same हूँगे
i.e. Patm.
So,
 $dP = 0$

From (1)

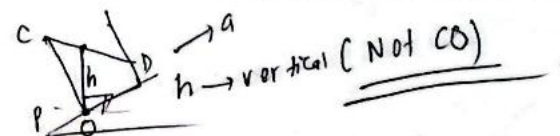
Vertical plane मा गति हिउँछ भने $a_y = 0 \Rightarrow \text{slope} = 0$ { Before After slope change हुँदैन }
Horizontal plane मा गति, $a_y = 0$ surface does not change

Inclined,

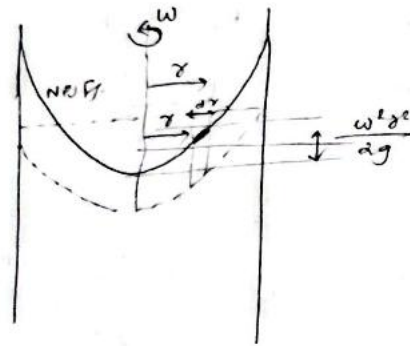


अगरि जादा liquid चढ्दा तब मात्र खोज्छ

Hydrostatic law : $P = \gamma h$
 $h \rightarrow$ vertical distance



$\Sigma F_x = F_I$
 $\frac{\partial p}{\partial x} dr dA = m a_x$
 $\frac{\partial p}{\partial x} dr dA = \rho (dA dr) (\omega^2 r)$
 $\frac{\partial p}{\partial x} = \rho \omega^2 r$
 $r \uparrow \quad p \uparrow$



Total change in pressure $dp = f(\text{space, time})$
 $dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial \theta} d\theta + \frac{\partial p}{\partial z} dz + \frac{\partial p}{\partial t} dt$
 $dp = \rho \omega^2 r dr - \gamma dz$
 $p = \frac{\rho \omega^2 r^2}{2} - \gamma z + C$

At 'D', $r=0, z=z_0, p=p_a$

$p_a = -\gamma z_0 + C$

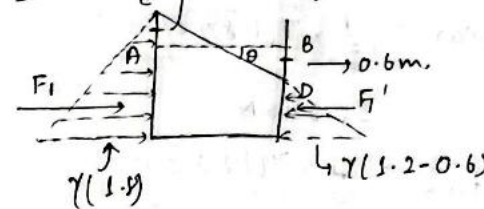
$C = p_a + \gamma z_0$

$p = p_a + \frac{\rho \omega^2 r^2}{2} - \gamma (z - z_0)$

Problems

1. Q0101.

case a)

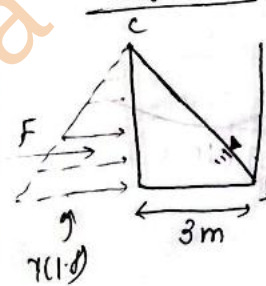


$F_1 - F_1' = m a$

$\text{or, } \frac{1}{2} \times \gamma (1.2) \times 1.8 - \frac{1}{2} \gamma (0.6) \times 0.6 = \frac{(1.2 \times 3 \times 1.5) \times \gamma}{\text{vol.}} \times a$
 $\Rightarrow (a)_c = 9.924 \text{ m/s}^2$

$\Delta F = F_1 - F_1'$
 $(V)_{\text{spill}} = 0 \text{ m}^3$

case (b)



$F = \frac{1}{2} \times \gamma (1.8) \times 1.8 \times 1.5$

$[F = 23.84 \text{ kN}]$ rear end horizontal

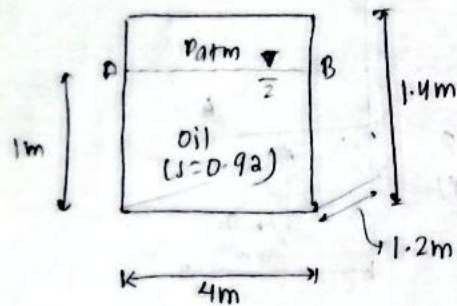
$F_{\text{bottom}} = \frac{1}{2} \times \gamma (1.8) \times 3 \times 1.5$
 $= 39.73 \text{ kN}$

$F_{\text{front end}} = 0 \text{ kN}$

Pressure intensities, $P_{\text{rear end}} = 23.84 \times (1.8 \times 1.5)$

$a = \frac{(\Delta F)_{\text{horizontal}}}{m} = \frac{23.84 \times 1000}{(1.2 \times 3 \times 1.5) \times 1000}$

2



$$\tan \theta = \frac{a}{g}$$

$$\Rightarrow \theta = 21.80^\circ$$

$$a_{ac} = \left(\frac{1}{2} \times (1.4 - y_1) \times 1.4 \right) \times 1.2$$

$$- \left(\frac{1}{2} \times (0.6) \times 0.6 \right) \times 1.2$$

$$\frac{(4 \times 1 \times 1.2) \times 1000}{\text{vol}^2}$$

$$= 1.962 \text{ m/s}^2$$

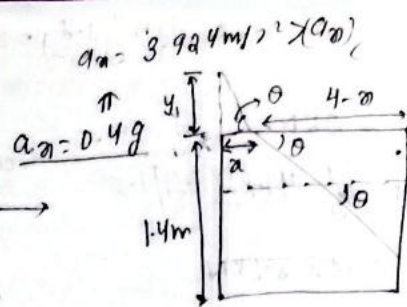
$$a = 2.76 \text{ m/s}^2$$

$$y_1 = (1.4 - 2.76) \tan(\theta) = 1.1 \text{ m} \quad 0.496 \text{ m}$$

$$P_{\text{rear bottom}} = \gamma h_1 = (0.92 \times 9810) \times (1.4 + 0.496)$$

$$= 17.11 \text{ kN/m}^2$$

$$P_{\text{front bottom}} = \gamma h_2 = (0.92 \times 9810) \times (1.4 - (4 - x) \tan \theta)$$



$$\text{For } m$$

$$(Vol)_{\text{before}} = (Vol)_{\text{after}}$$

$$1.2 \times 4 \times 1 = (0.4 \times 4 \times 1.2) + \frac{1}{2} \times (4 - x)^2 \tan \theta$$

$$(4 - x) = \sqrt{1.536}$$

$$\text{or } 4 - x = \pm 1.24$$

$$(+ve) \quad x = 2.76 \text{ m}$$

$$(-ve) \quad x = 5.24 \text{ m} > 4 \text{ m}$$

2

21.80°

Free-space

$$(V)_{\text{before}} = (V)_{\text{after}}$$

$$(1.2 \times 4 \times 0.4)$$

$$= \frac{1}{2} \times x \times x \tan \theta \times 1.2$$

$$\Rightarrow x = 3.09 \text{ m} \quad 2.828 \text{ m}$$

$$4 - x = 0.91 \text{ m} \quad 1.17 \text{ m}$$

$$y_1 = 0.91 \tan \theta = 0.46 \text{ m} \quad 0.47 \text{ m}$$

$$P_{\text{rear}} = 0.92 \times 9810 \times \left(\frac{1.4 + 0.47}{2} \right)$$

$$= 16.87 \text{ kN/m}^2$$

$$P_{\text{front}} = 0.92 \times 9810 \times \left(\frac{1.4 - 2.828 \tan \theta}{2} \right)$$

$$= 2.43 \text{ kN/m}^2$$

$$P_{\text{bottom}} = 0.92 \times 9810 \times \left(\frac{1.87 + 0.268}{2} \right) \times \frac{4}{3} \times 1.2$$

$$P_{\text{top}} = \gamma(y_1) = 4.24 \text{ kN/m}^2$$

$$F_{\text{rear}} = (P_{\text{top}})_{\text{PD}} \times \text{width} \times 0.47$$

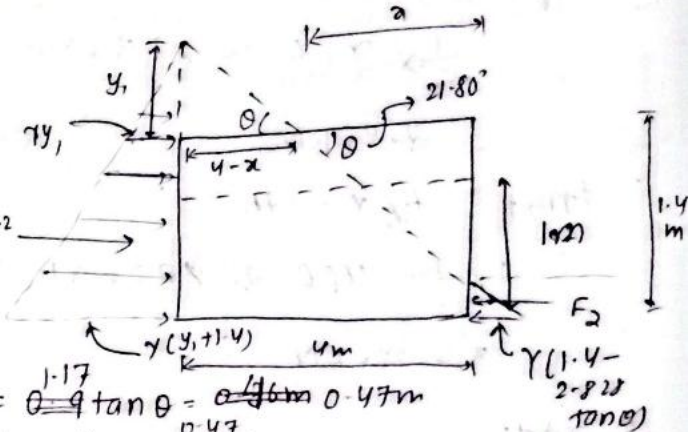
$$= 0.92 \times 9810 \times \left(\frac{y_1 + (y_1 + 1.4)}{2} \right) \times \frac{1.4}{3} \times 1.2$$

$$= 17.74 \text{ kN}$$

$$F_{\text{bottom}} = (P_{\text{top}})_{\text{PD}} \times \text{width}$$

$$= 0.92 \times 9810 \times \left(\frac{1.87 + 0.27}{2} \right) \times \frac{4}{3} \times 1.2$$

$$= 46.35 \text{ kN}$$



$$F_{top} = (A)_{\Delta} \times \text{width}$$

$$= \frac{1}{2} \times \gamma (0.47) \times 1.17 \times 1.2$$

$$= 2.98 \text{ kN}$$

$$F_{front} = A_{\Delta} \times \text{width}$$

$$= \frac{1}{2} \times \gamma (0.27) \times 0.27 \times 1.2$$

$$= 0.395 \text{ kN}$$

PD
liquid surface
bottom surface
free surface

© contd:

Verify forces are in eq.

① $F_{rear} - F_{front}$

$$= 17.34 \text{ kN}$$

② $m a_x = \frac{(1 \times 4 \times 1.2)}{\text{vol}} \times \frac{0.92 \times 1000}{s} \times \frac{0.4 \times 9.81}{a_x}$

$$= 17.33 \text{ kN}$$

$$\approx 17.34 \text{ kN}$$

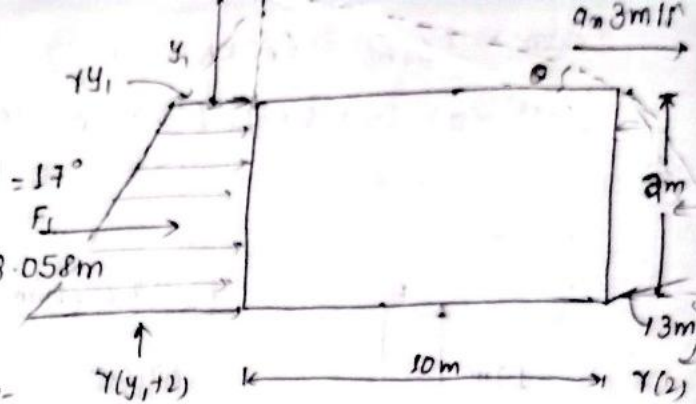
verified

③

$$\tan \theta = \frac{a_x}{g}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{3}{9.81} \right) = 17^\circ$$

$$y_1 = 10 \tan \theta = 3.058 \text{ m}$$



(a) Pressure intensities:-

$$(P_{bottom})_{front} = \gamma \times 2 = 19.62 \text{ kN/m}^2$$

$$(P_{bottom})_{rear} = \gamma \times (3.058 + 2) = 49.62 \text{ kN/m}^2$$

$$(P_{top})_{rear} = \gamma (y_1) = 29.99 \text{ kN/m}^2$$

$$(P_{top})_{front} = \gamma \times 0 = 0 \text{ kN/m}^2$$

④

$$F_{rear} = (A_{trap})_{PD} \times \text{width}$$

$$= \gamma (0.47 + 0.4) = \frac{\gamma (3.058 + 3.058 + 2) \times 2 \times 3}{2}$$

$$= 238.85 \text{ kN}$$

$$F_{front} = (A_{\Delta})_{PD} \times \text{width} = \frac{1}{2} \times \gamma (2) \times 2 \times 3$$

$$= 58.86 \text{ kN}$$

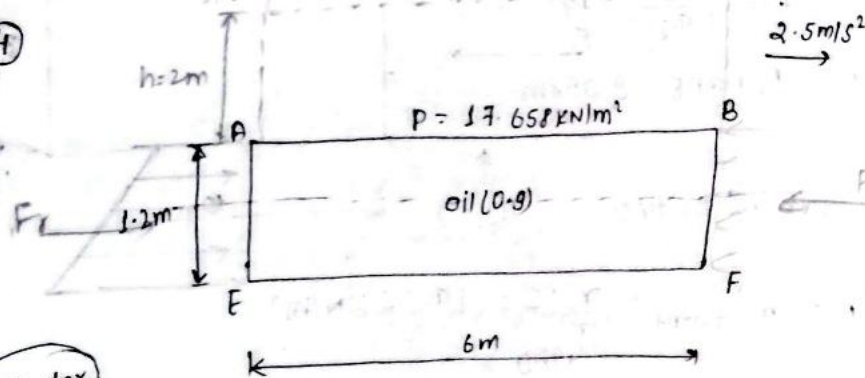
$$F_{bottom} = (A_{trap})_{PD} \times \text{width} = \left\{ \gamma \frac{(y_1 + 2) + 2}{2} \times 10 \right\} \times 3$$

$$F_{top} = (A_{\Delta}) \times \text{width} = \frac{1}{2} \times (\gamma y_1) \times 10 \times 3 = 449.98 \text{ kN}$$

$$F_{\text{rear}} - F_{\text{front}} = 180 \text{ kN}$$

$$m a_m = (2 \times 10 \times 3) \times 1000 \times 3 = 180 \text{ kN}$$

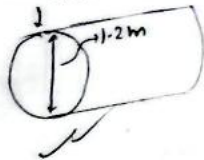
(4)



cylinder

$$F_{AE} = ? \quad F_{BF} = ?$$

AE/BF



Extra pressure given $\rightarrow$ corr. head
 $P = 17.658 \text{ kN/m}^2 = \gamma h$

$$0.9 \times 9810$$

$$\Rightarrow h = 2 \text{ m} \rightarrow \text{यदि माछि गहिरा सुक उर्ने}$$

$$\tan \theta = \frac{a_m}{g} = 2.5$$

$$\Rightarrow \theta = 14.3^\circ$$

$$y_1 = 6 \tan(14.3^\circ) = 1.53 \text{ m}$$

Not rectangular : Use formula.

$$F_x = \gamma A (\bar{y}) = 0.9 \times 9810 \times \left(\frac{\pi \times 1.2^2}{4} \right) \times \left(1.528 + 2 + \frac{1.2}{2} \right)$$

$$= 43.22 \text{ kN}$$

$$F_f = \gamma A (\bar{y})$$

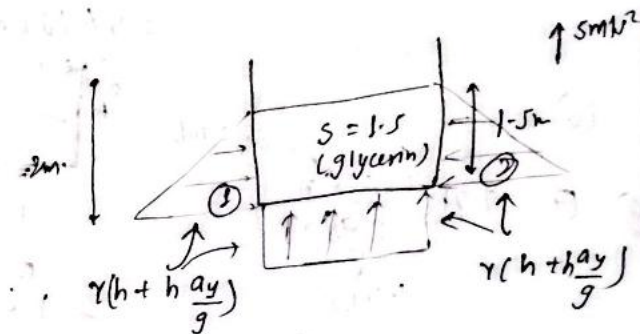
$$= (0.9 \times 9810) \times \left(\frac{\pi \times 1.2^2}{4} \right) \times \left(2 + \frac{1.2}{2} \right)$$

$$= 25.96 \text{ kN}$$

$$F_{\text{bottom}} = \gamma A (\bar{y})$$

varies so, integration req. taking small vertical strip.

(5)



(a)

$$P_{(1)} = \gamma \left(h + h \frac{a_y}{g} \right)$$

$$= 1.5 \times 9810 \times \left(1.5 + 1.5 \times \frac{5}{9.81} \right)$$

$$= 35.54 \text{ kN/m}^2$$

$$P_{(2)} = \gamma \left(h + h \frac{a_y}{g} \right)$$

$$= 35.54 \text{ kN/m}^2$$

$$F_1 = F_2 = \frac{1}{2} \times (35.54) \times 1.5 \times 2 = 53.31 \text{ kN}$$

$$F_{\text{bottom}} = \left\{ \gamma \left(h + h \frac{a_y}{g} \right) \times 2 \right\} \times 2$$

$$= 142.16 \text{ kN}$$

② Downward accelⁿ

$$p = \gamma \left(h - h \frac{a_y}{g} \right)$$

$$= 11.54 \text{ kN/m}^2$$

$$F_1 = F_2 = \frac{1}{2} \left(\gamma \left(h - h \frac{a_y}{g} \right) \times 1.5 \right) \times 2$$

$$= 17.31 \text{ kN}$$

$$F_{\text{bottom}} = \gamma \left(h - h \frac{a_y}{g} \right) \times 2 \times 2$$

$$= 46.16 \text{ kN}$$

⑥

(a) $\theta = \tan^{-1} \left(\frac{a_n}{a_y + g} \right)$

$$= 15.042^\circ$$

$$= 15^\circ 2'$$

Not-mentioned
NO spill $\rightarrow$ passes thr. centre of original surface

(b) $\tan \theta = \frac{x}{2.5} \Rightarrow x = 0.67 \text{ m}$

(P_{bottom})_{rear} = $\frac{\gamma \times (2.67)}{h} \left(1 + \frac{1.8}{9.81} \right)$

$$= 24.8 \text{ kN/m}^2$$

(P_{bottom})_{front} = $\frac{\gamma \times (2.067)}{h} \left(1 + \frac{1.8}{9.81} \right)$

$$= 12.35 \text{ kN/m}^2$$

★ Vertical accelⁿ $\uparrow$
★ at static press. at bottom after

③ $F_{\text{front}} = \left\{ \frac{1}{2} \times \gamma (1.33) \left(1 + \frac{1.8}{9.81} \right) \times \dots \right\} \times \text{width}$

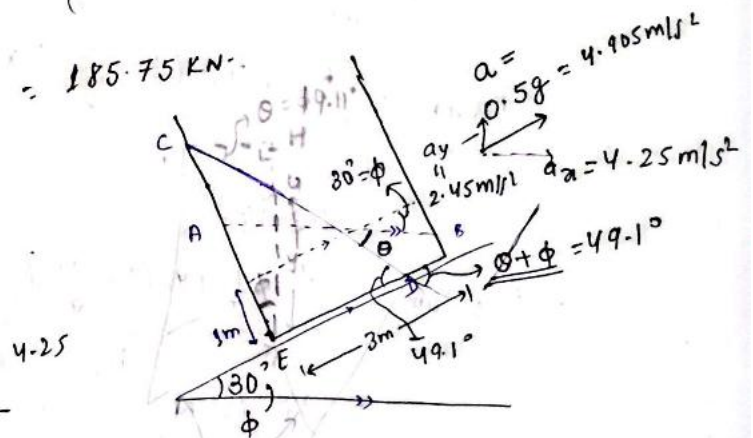
$$= 16.43 \text{ kN}$$

$$F_{\text{rear}} = \left\{ \frac{1}{2} \times \gamma (2.67) \left(1 + \frac{1.8}{9.81} \right) \times 2.67 \right\} \times 2$$

$$= 66.2 \text{ kN}$$

$$F_{\text{bottom}} = (A_{\text{trap}}) \times \text{width}$$

$$= \left\{ \gamma \left\{ \left(2.67 \left(1 + \frac{1.8}{9.81} \right) + 1.33 \left(1 + \frac{1.8}{9.81} \right) \right) \times 5 \right\} \times 2 \right\}$$



⑦ 3m long, 1m wide,
3m deep
water upto depth
1m.

(a) $\tan \theta = \frac{a_n}{a_y + g}$

$$= \frac{4.25}{2.45}$$

$$\theta = 19.1^\circ$$

(b) Assuming: No spillage condition

$\therefore (\text{Vol})_{\text{before accel}} = (\text{Vol})_{\text{after accel}}$

or, $(1 \times 1 \times 3) = \frac{1}{2} \times CE \times ED \times 1 \text{ m}$

$$9 = \frac{1}{2} \times ED^2 \tan(49.1^\circ) \times 1$$

$$\Rightarrow ED = \frac{3.95 \text{ m}}{2} = ED = 2.28 \text{ m}$$

$$CE = ED \tan(19.1^\circ)$$

$$CE = 2.63 \text{ m}$$

$$y_1 = EG = EH - EG$$

$$= EH - HC \tan(19.1^\circ)$$

$$= EH - CE \sin(30^\circ) \tan(19.1^\circ)$$

$$= \frac{CE \cos(30^\circ)}{2.63} - CE \sin(30^\circ) \tan(19.1^\circ)$$

$$= 1.82 \text{ m}$$

$$P_{\text{bottom}} = \gamma \left(y_1 + y_1 \frac{ay}{g} \right) = 22.31 \text{ kN/m}^2$$

$$F_{\text{rear}} = \frac{1}{2} \times \left(\gamma y_1 \left(1 + \frac{ay}{g} \right) \right) \times CE \times 1$$

$$= 29.34 \text{ kN}$$

$$F_{\text{bottom}} = \frac{1}{2} \times \left[\gamma y_1 \left(1 + \frac{ay}{g} \right) \right] \times ED \times 1$$

$$= 25.43 \text{ kN}$$

Next to show forces are in eq^m

Balance vertical
& horzn^l force

$$F_2 = F_T = m a$$

$$= \frac{29.43}{9.81} \times 0.5 \times 9.81$$

$$= 14.72 \text{ kN}$$

$$W = \gamma(V) = 29.43 \text{ kN}$$

$$\begin{aligned} \text{Horizontal} \quad \left\{ \begin{aligned} F_{\text{right}} &= 29.34 \text{ kN} \\ F_{\text{left}} &= F_2 + W \sin(30^\circ) \\ &= 14.72 + 29.43 \sin(30^\circ) \\ &= 29.43 \text{ kN} \end{aligned} \right. \end{aligned}$$

$$\begin{aligned} \text{Vertical} \quad \left\{ \begin{aligned} F_{\text{down}} &= \frac{W}{\cos \theta} \\ &= 25.48 \text{ kN} \\ F_{\text{up}} &= F_{\text{bottom}} = 25.43 \text{ kN} \end{aligned} \right. \end{aligned}$$

8) Solⁿ

$$\frac{FS}{V_{\text{spillage}}} = \frac{(V_{\text{before}})_{\text{rot}}}{(V_{\text{after}})_{\text{rot}}}$$

$$= \frac{(V_{\text{after rot}})_{\text{para}}}{(V_{\text{before rot}})_{\text{cylinder}}} \frac{F_1}{F_2}$$

$$= \left(\frac{1}{2} \times \pi \times (7.5)^2 \times 100 \right) - \left(\pi \times (7.5)^2 \times 30 \right)$$

$$= 8835.73 - 4241.53 = 4594.2$$

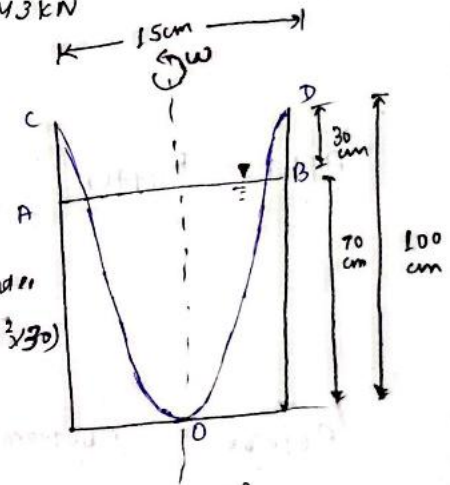
$$= 3534.3 \text{ cm}^3$$

$$= 3.53 \times 10^{-3} \text{ m}^3$$

$$\text{Also, (From datum of parabola bottom)} \quad Z = \frac{100}{10^{-2}} = \frac{\omega^2 \times r^2}{2g}$$

$$\Rightarrow \omega = 59.06 \text{ rad/s}$$

$$\text{Number of rotations per minute} \quad \omega = \frac{2\pi N}{60} \Rightarrow N = 563.98 \text{ rpm}$$



(For CD,
 $r = R$
 $= 7.5 \text{ cm}$)

③ Pressure intensities:

$$P_{\text{bottom}} = 0.8 \text{ atm} \quad (7 \times 10^5)$$

$$P_{\text{bottom}} \text{ at } 2 \text{ m depth} = 7 \times 10^5 \text{ N/m}^2$$

$$= 9.8 \text{ atm}$$

④ Pressure force:

Before: $F_{\text{side}} = 7 \times 10^5 \times 1 \text{ m} \times 1 \text{ m}$
 $= 7 \times 10^5 \text{ N}$
 $F_{\text{bottom}} = 9.8 \times 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$
 $= 9.8 \times 10^5 \text{ N}$

After: $F_{\text{side}} = 7 \times 10^5 \text{ N}$
 $F_{\text{bottom}} = 9.8 \times 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$
 $= 9.8 \times 10^5 \text{ N}$

Before: $F_{\text{bottom}} = 7 \times 10^5 \text{ N}$
 $F_{\text{side}} = 9.8 \times 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$
 $= 9.8 \times 10^5 \text{ N}$

After: $F_{\text{bottom}} = 7 \times 10^5 \text{ N}$
 $F_{\text{side}} = 9.8 \times 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$
 $= 9.8 \times 10^5 \text{ N}$

After: $F_{\text{bottom}} = F_1 + F_2 \rightarrow 0$ (bottom center)
 $F_1 = \frac{\pi}{4} \times 1000 \times (0.5 \text{ m})^2 \times 9.8$
 $F_2 = \frac{\pi}{4} \times 1000 \times (0.5 \text{ m})^2 \times 9.8$
 $= 0.0875 \text{ N}$

⑤ Pressure in pressure force before and after
 $\Delta F_{\text{side}} = 1.2 \times 10^5 \text{ N} - 1.1 \times 10^5 \text{ N}$
 $\Delta F_{\text{bottom}} = 0.8 \times 10^5 \text{ N} - 0.7 \times 10^5 \text{ N}$

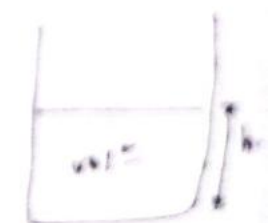
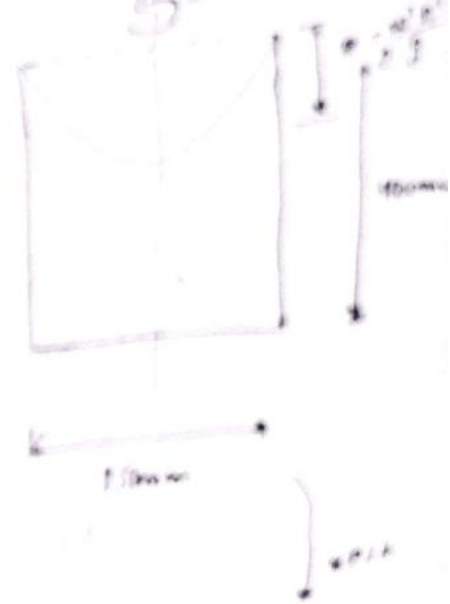
⑥ Pressure force
 $F_{\text{side}} = 1.2 \times 10^5 \text{ N}$
 $F_{\text{bottom}} = 0.8 \times 10^5 \text{ N}$

⑦ Pressure force
 $F_{\text{side}} = 1.2 \times 10^5 \text{ N}$
 $F_{\text{bottom}} = 0.8 \times 10^5 \text{ N}$

Volume = $\frac{1}{2} \times \pi \times R^2 \times h$
 $= \frac{1}{2} \times \pi \times (0.5 \text{ m})^2 \times 1 \text{ m}$
 $= 0.39 \text{ m}^3$

Volume remaining = (Vol) before - (Vol) spilled
 $= 4.24 \times 10^{-3} \text{ m}^3$

Volume remaining = $\pi R^2 h$
 $\text{depth of water } (h) = 0.24 \text{ m}$



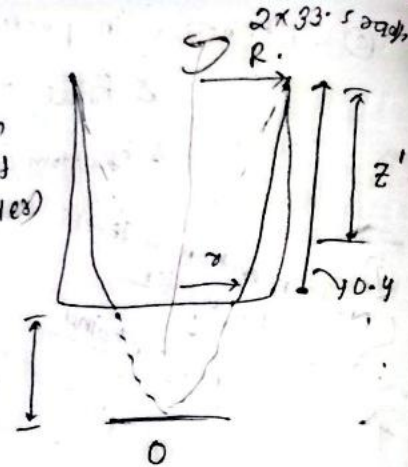
(b)

speed doubled

$$z' = \frac{\omega'^2 R^2}{2g} = 1.287 \text{ m}$$

($>$ height of cylinder)

lower than bottom



$$z_y = 1.287 - 0.4 = 0.887 \text{ m}$$

$$z_r = \frac{\omega^2 r^2}{2g} \quad r = 0.062 \text{ m}$$

~~38.76 x 10~~Uncovered area = πr^2 → in bottom.

$$(Vol)_{\text{spilled}} = (V_{\text{parabola}})_{\text{whole}} - (V_{\text{parabola}})_{\text{small}}$$

$$= \left[\frac{1}{2} \times \pi \times (75 \times 10^{-3})^2 \times 1.287 \right] - \left[\frac{1}{2} \times \pi \times (0.062)^2 \times 0.887 \right]$$

$$= 0.00601 \text{ m}^3$$

$$= 6.01 \times 10^{-3} \text{ m}^3$$

$$(V)_{\text{before}} = \pi R^2 \times 0.4 = 7.07 \times 10^{-3} \text{ m}^3$$

$$(V)_{\text{remaining}} = (7.07 - 6.01) \times 10^{-3} = 1.06 \times 10^{-3} \text{ m}^3$$

$$V_{\text{remaining}} = \pi R^2 \times h$$

$$h = 0.0596 \text{ m}$$

(10)

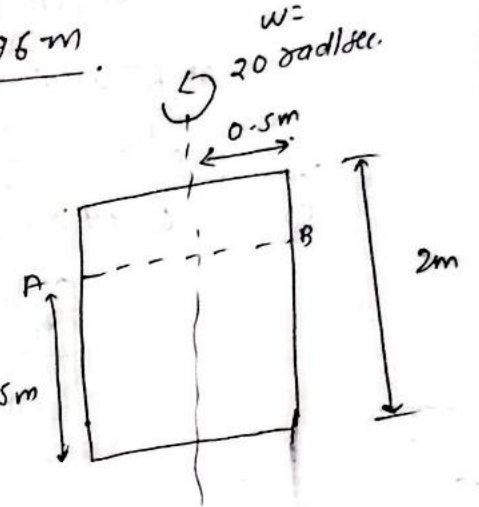
$$Z = \frac{\omega^2 R^2}{2g} = 5.1 \text{ m} (> 2 \text{ m})$$

$$ht \rightarrow 2 \text{ m} < 5.1 \text{ m}$$

So, bottom is uncovered as well as water goes in top side.

(A volume)

If whole bottom just covered $(r=0)$ No prob



(a)

$$r_1 \text{ \& } r_2 \rightarrow ?$$

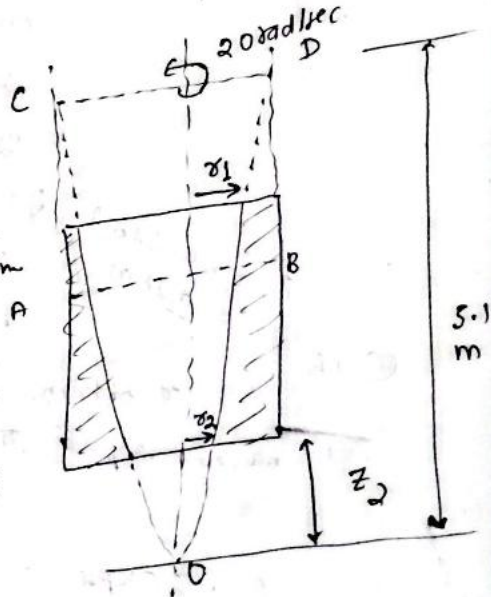
$$z_1 = \frac{\omega^2 r_1^2}{2g} = 20.38 r_1^2 \quad \text{--- (I)}$$

$$z_2 = \frac{\omega^2 r_2^2}{2g} = 20.38 r_2^2 \quad \text{--- (II)}$$

$$z_1 = z_2 + 2$$

$$20.38 r_1^2 = 20.38 r_2^2 + 2$$

$$r_1^2 - r_2^2 = 0.098 \quad \text{--- (III)}$$



(FS) (Vol) before rotation = (Vol) after rotation.

$$\pi R^2 \times (2-1.5) = \frac{1}{2} \pi x_1^2 x_2 - \frac{1}{2} \pi x_2^2 x_2$$

$$\pi \times 0.5^2 \times 0.5 = 20.38 x_1^2 - 20.38 x_2^2$$

$$0.393 = 20.38 x_1^2 - 20.38 x_2^2$$

$$x_1^4 - x_2^4 = 0.0123 \quad \text{--- (iv)}$$

From (iii) & (iv)

$$x_1^2 + x_2^2 = 0.1255 \quad \text{--- (v)}$$

From (ii) & (v)

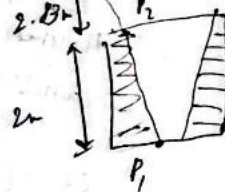
$$x_1^2 = 0.1118$$

$$x_2^2 = 0.01375$$

$$x_1 = 0.334 \text{ m}$$

$$x_2 = 0.117 \text{ m}$$

$$Z_2 = 20.38 x_2^2 = 0.28 \text{ m}$$



$$Z_1 = 20.38 x_1^2 = 0.28 \text{ m}$$

(A) uncovered bottom = $\pi x_2^2 = 0.043 \text{ m}^2$

(A) uncovered top = $\pi (x_1^2) = 0.35 \text{ m}^2$

$P = 601094 = \frac{60 \times 1000}{9810} \times 6.11 \text{ m of water (above)}$

$P_1 = P_{\text{side bottom}} = 6.11 + 2.83 + 2 = 10.94 \text{ m of water}$

$P_2 = P_{\text{top}} = 6.11 + 2.83 = 8.94 \text{ m}$

$(R) = 6.11 \text{ m of water}$
 (center of bottom) (center of top)

$P_{\text{side}} = \gamma \times h = 9810 \times 6.11 = 59.9 \text{ kN/m}^2$

$(P_{\text{bottom}})_{\text{centre}} = \gamma h' = 9810 \times 10.94 = 107.32 \text{ kN/m}^2$

$(P_{\text{top}})_{\text{edge}} = \gamma h_{\text{top edge}} = 9810 \times (6.11 + 2.83) = 87.7 \text{ kN/m}^2$

$(P_{\text{top}})_{\text{centre}} = \gamma h_{\text{top centre}} = 9810 \times 6.11 = 59.9 \text{ kN/m}^2$

(c) $F_{\text{side}} = \gamma A \bar{y} = 9810 \times \left\{ (2\pi \times 1) \times 2 \right\} \times \left(6.11 + 2.83 + \frac{2}{2} \right)$

$= 612.68 \text{ kN}$

$F_{\text{bottom}} = \gamma A \bar{y} = 9810 \times \left\{ \frac{\pi \times 1^2}{2} \right\} \times (10.94)$

$$= 9810 \times 0.7424 \times 10.94$$

$F_{\text{bottom}} = F_1 + F_2$

$(F_1) = \gamma \times (\pi \times 0.5^2) \times 6.11 = 47.08 \text{ kN}$
 above parabola

$(F_2)_{\text{below CD}} = \frac{8 \omega^2 \pi}{4} [x^4]$
 $\rightarrow 0.5$
 $\rightarrow 0.117$
 $= 24 \text{ kN} + 19.58 \text{ kN}$

$F_{\text{bottom}} = F_1 + F_2 = 47.08 + 43.58 = 90.66 \text{ kN}$

$$F_{top} = \frac{\rho \omega^2 \pi}{4} [r^4]_{r_1}^R + 47.08$$

$$= 15.73 + 47.08$$

$$= 62.81 \text{ kN}$$

(11)

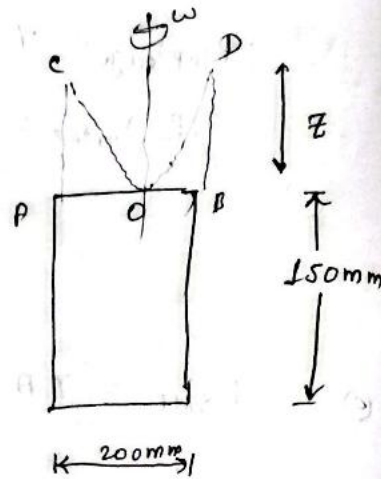
$$N = 2000 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = 20.94 \text{ rad/s}$$

$$z = \frac{\omega^2 r^2}{2g}$$

$$= \frac{(20.94)^2 \times (0.1)^2}{2 \times 9.81}$$

$$= 0.223 \text{ m}$$



$$(P_{bottom})_{edge} = \gamma (0.15 + z) = 9.66 \text{ kN/m}^2$$

$$(P_{bottom})_{centre} = \gamma (0.15) = 1.47 \text{ kN/m}^2$$

$$(P_{top})_{centre} = 0 \text{ kN/m}^2$$

$$(P_{top})_{edge} = \gamma (z) = 2.19 \text{ kN/m}^2$$

$$F_{bottom} = F_1 + F_2$$

$$= \frac{\rho \omega^2 \pi}{4} [r^4]_{r_1}^{r_2} + \gamma A \bar{y}$$

$$= 34.44 + 9810 \times \pi \times 0.1^2 \times 0.15$$

$$= 80.66 \text{ kN}$$

$$F_{top} = \frac{\rho \omega^2 \pi}{4} [r^4]_{r_1}^{r_2}$$

$$= 34.44 \text{ kN}$$

$$F_{side} = \gamma A \bar{y}$$

$$= 9810 \times \left\{ (2\pi \times 0.1) \times 0.15 \right\} \times \left(\frac{0.15}{2} + 0.223 \right)$$

$$= 275.5 \text{ kN}$$

(12) Sol:-

$$(P_{bottom})_{max} = 1177.2 \text{ kN/m}^2$$

$$z = \frac{\omega^2 r^2}{2g} = 0.051 \text{ m}$$

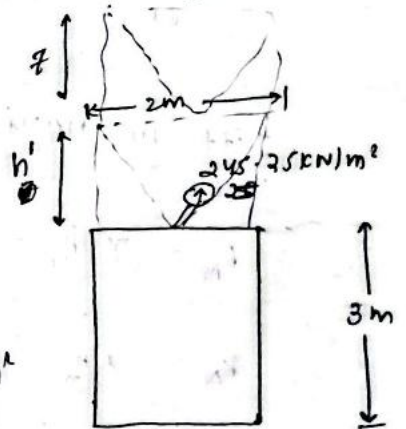
$$(P_{bottom})_{max} = 1177.2 \text{ kN/m}^2$$

$$\text{or, } \gamma (z + h' + 3) = 1177.2 \text{ kN/m}^2$$

$$9.81 \times 1.6$$

$$\Rightarrow 0.051 (\omega_{max})^2 = 75 - 15.625 - 3$$

$$\omega_{max} = 33.247 \text{ rad/s}$$



$$p = 245.25 = (9810 \times 1.6) \times h'$$

$$h' = 15.625 \text{ m}$$

$$\omega = \frac{2\pi N}{60}$$

$$N = \frac{\omega \times 60}{2\pi}$$

$$N_{max} = 317.5 \text{ rpm}$$

(13)

$$N = 60 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = 6.283 \text{ rad/s}$$

$$z_1 = \frac{\omega^2 r_1^2}{2g}$$

$$= 0.181 \text{ m}$$

$$z_2 = \frac{\omega^2 r_2^2}{2g} = 0.0453 \text{ m}$$

No spilling condition,

★ Ht. before rotation = Ht. After rotation

$$\text{or, } (15 + 15) \times 10^{-2} = \frac{z_1 + z_2 + 2z_0}{2}$$

$$\text{or, } 30 \times 10^{-2} = 0.181 + 0.0453 + 2z_0$$

$$\Rightarrow \boxed{z_0 = 0.0368 \text{ m}}$$

$$\text{Diff. in liquid level} = z_1 - z_2$$

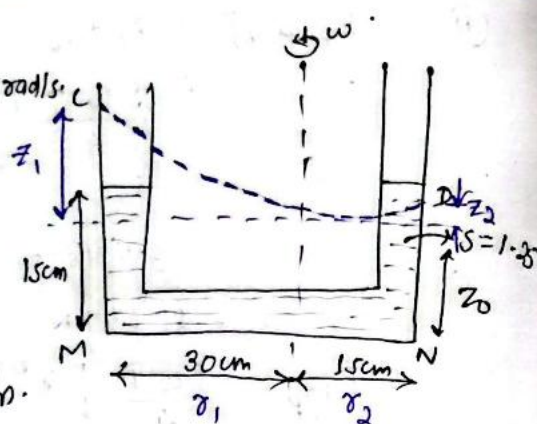
$$= 13.57 \text{ cm}$$

$$P_M = \gamma h_1 = (1.25 \times 9810) (z_0 + z_1)$$

$$= 2.67 \text{ kPa}$$

$$P_N = \gamma h_2 = (1.25 \times 9810) (z_0 + z_2)$$

$$= 1.007 \text{ kPa}$$



Chapter-7

Fundamental of fluid flow

Numericals

(1) Continuity equation

$$(a) \quad u = 5x^2 + 2xy; \quad w = 2z^3 - 4xy - 2yz$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\text{or, } \frac{\partial (5x^2 + 2xy)}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial (2z^3 - 4xy - 2yz)}{\partial z} = 0$$

$$\text{or, } (10x + 2y) + \frac{\partial v}{\partial y} + (6z^2 - 2y) = 0$$

$$\text{or, } \int \partial v = \int (10x + 6z^2) \partial y$$

$$v = -10xy - 6z^2y + C$$

★ $\int w \cdot r \cdot t \cdot y$
 $\Rightarrow$ so, x & z const. as well
 along with other,
 $C \neq f(y)$
 $C = f(x, z)$

$$(b) \quad u = x^3 + y^2 + 2z^2; \quad v = -x^2y - yz - xy$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\text{or, } 3x^2 - x^2 - z - x + \frac{\partial w}{\partial z} = 0$$

$$\text{or, } \partial w = (-2x^2 + z - x) \partial z$$

$$w = -2x^2z + \frac{z^2}{2} - xz + f(x, y)$$

© $u = \log(y^2+z^2)$; $v = \log(x^2+z^2)$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

or, $\partial w = 0$

$\Rightarrow w = C$ (By symmetry can be $\log(x^2+y^2)$)

(d) $u = \frac{-2xyz}{(x^2+y^2)^2}$; $w = \frac{y}{x^2+y^2}$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^2)^2 \cdot (-2yz) + 2xyz \cdot 2x \cdot 2(x^2+y^2)}{\{(x^2+y^2)^2\}^2}$$

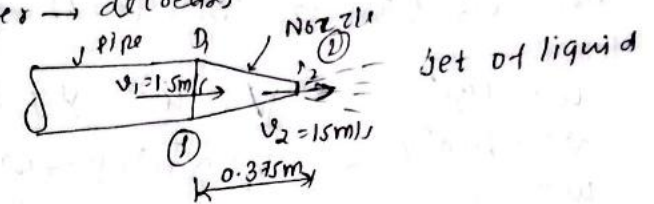
$$= \frac{-2x^3yz - 2y^3z + 8x^3yz + 8x^2y^2z}{(x^2+y^2)^3}$$

$$= \frac{6x^2yz - 2y^3z}{(x^2+y^2)^3}$$

②

Nozzle

velocity : 1.5 m/s $\rightarrow$ 15 m/s
 diameter $\rightarrow$ decreases



acceleration = $v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$
 convective accⁿ (w.r.t. space) local (or temporal) (w.r.t. time)

$$a_{\text{convective}} = v \frac{\partial v}{\partial s} = v \times \frac{(15-1.5)}{0.375}$$

$$a_{\text{con}} = 36v \quad \text{--- (1)}$$

For section ①

$$v_1 = 1.5 \text{ m/s}$$

$$a_{\text{conv}} = 54 \text{ m/s}$$

For section ②

$$v_2 = 15 \text{ m/s}$$

$$a_{\text{conv}} = 540 \text{ m/s}$$

Q Sol:-

velocity vector,

$$V = 4x^3 \vec{i} - 10x^2 y \vec{j} + 2t \vec{k}$$

At, (2, 1, 3) at time (t=1)

Here,

$$\left. \begin{aligned} u &= 4x^3 \\ v &= -10x^2 y \\ w &= 2t \end{aligned} \right\} \vec{V} = u \vec{i} + v \vec{j} + w \vec{k}$$

velocity, $V = 32 \vec{i} - 40 \vec{j} + 2 \vec{k}$

$$V = \sqrt{32^2 + (-40)^2 + 2^2} = 51.26 \text{ units.}$$

Acceleration,

$$a = i a_x + j a_y + k a_z$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\begin{aligned} a_x &= v \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \\ &= 32 \times (12 \times 2^2) + (-40) \times (0) + (2) \times (0) + (0) \\ &= 1536 \text{ units} \end{aligned}$$

$$\begin{aligned} a_y &= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \\ &= (32)(-20 \times 2 \times 1) + (-40)(-10 \times 2^2) + 2 \times 0 + 0 \\ &= 320 \text{ units} \end{aligned}$$

$$\begin{aligned} a_z &= u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \\ &= (32) \times 0 + (-40) \times 0 + 2 \times 0 + 2 \\ &= 2 \text{ units} \end{aligned}$$

$$a = 1536 \vec{i} + 320 \vec{j} + 2 \vec{k}$$

$$a = 1568.98 \text{ units.}$$

Rotation (ω)

$$\begin{aligned} \omega_x &= \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \\ &= \frac{1}{2} (0 - 0) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \omega_y &= \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \\ &= \frac{1}{2} (0) \\ &= 0 \end{aligned} \quad \left| \quad \begin{aligned} \omega_z &= \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \\ &= \frac{1}{2} (-20 \times 2 \times 1) \\ &= -20 \end{aligned} \right.$$

Thus,

$$\omega = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

$$\omega = 0 \vec{i} + 0 \vec{j} - 20 \vec{k}$$

$$\omega = 20 \text{ units.}$$

Vorticity (ξ)

$$\xi_x = 2\omega_x = 0$$

$$\xi_y = 2\omega_y = 0$$

$$\xi_z = 2\omega_z = -40$$

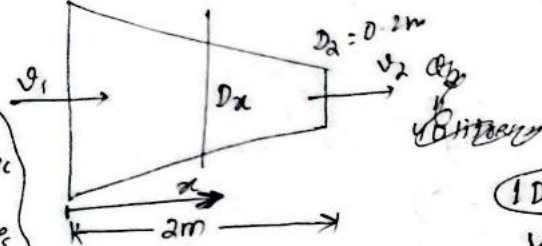
$$\therefore \xi = 0 \vec{i} + 0 \vec{j} - 40 \vec{k}$$

$$\xi = 40 \text{ units.}$$

(4) Solⁿ:

Initially
Discharge
 Φ_1
" 20 liter/sec
" $20 \times 10^{-3} \text{ m}^3/\text{sec}$

$$D_1 = 0.4 \text{ m}$$



1D-flow
 $v = u$

$$a_{\text{conv}} = v \frac{\partial v}{\partial s}$$

At distance, 'x'

Dia decreasing change
avg. $\left(\frac{0.4 - 0.2}{2} \right) x$

$$D_2 = 0.4 - \left(\frac{0.4 - 0.2}{2} \right) x$$

$$D_2 = 0.4 - 0.1x$$

$$Q = A v = \frac{\pi D^2}{4} \cdot v$$

$$v_x = \frac{4Q}{\pi D_2^2} = \frac{4Q/\pi}{(0.4 - 0.1x)^2}$$

At middle of pipe
 $x = 1 \text{ m}$

$$v = u_x = \frac{4Q/\pi}{(0.4 - 0.1x)^2} = \frac{14.15Q}{(0.4 - 0.1x)^2} = 0.283 \text{ m/s}$$

$$\frac{\partial v}{\partial s} = \frac{\partial u_x}{\partial x} = \frac{4Q}{\pi} (-2) (0.4 - 0.1x)^{-3} \times (-0.1)$$

For $Q = 20 \times 10^{-3} \text{ m}^3/\text{s}$ & $x = 1 \text{ m}$

$$\frac{\partial v}{\partial s} = 0.189 / \text{m}$$

$$a_{\text{conv}} = v \frac{\partial v}{\partial s} = 0.0534 \text{ m/s}^2$$

Next

Next, Q not const.

Rate of flow changes from 20 L/s to 40 L/s in 30 sec.
Total acc = at middle of pipe in 15th sec.
 $L = 1 \text{ m}$

$$a = v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

$$Q_t = 20 + \left(\frac{40 - 20}{30} \right) t = 20 + 0.67t$$

For $t = 15 \text{ sec}$.

$$Q_{15} = 30 \text{ liter/sec} = 0.03 \text{ m}^3/\text{sec}$$

$$\frac{\partial v}{\partial s} = \frac{4Q}{\pi} (-2) \frac{(0.4 - 0.1x)^{-3}}{0.1} = 0.283 / \text{sec}$$

$$a = v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

$$v = \frac{4Q/\pi}{(0.4 - 0.1x)^2} = 0.424 \text{ m/s}$$

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial t} \left(\frac{4Q/\pi}{(0.4 - 0.1x)^2} \right)$$

$$= \frac{4/\pi}{(0.4 - 0.1x)^2} \times \frac{\partial Q}{\partial t} \quad \text{changes with time}$$

$$= \frac{4/\pi}{(0.4 - 0.1x)^2} \times 0.67 \times 10^{-3} = 0.0095 \text{ m/s}^2$$

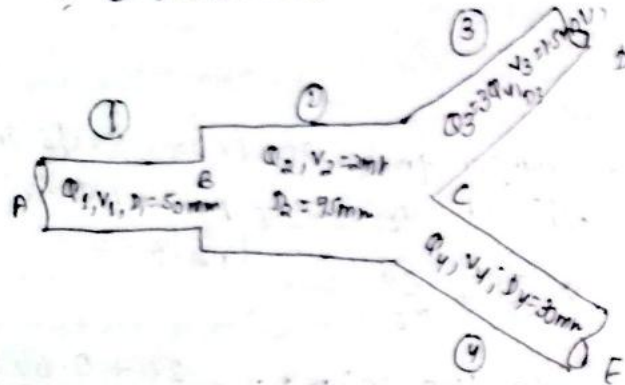
Then,

$$a = v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

$$= 0.424 \times 0.283 + 0.0095$$

$$= 0.1295 \text{ m/s}^2$$

⑤ Solⁿ



mass conservation

↳ continuity eqⁿ

For Section ① & ②

$$A_1 v_1 = A_2 v_2$$

$$\text{or } \frac{\pi}{4} D_1^2 \times v_1 = \frac{\pi}{4} \times D_2^2 \times v_2$$

$$v_1 = 4.5 \text{ m/s}$$

$$Q_1 = A_1 v_1 = \frac{\pi}{4} \times (50 \times 10^{-3})^2 \times 4.5 = Q_2 = 0.00884 \text{ m}^3/\text{s}$$

$$Q_1 = Q_2$$

Some mass has been divided into two sections from C,

So,

$$Q_1 = Q_2 = Q_3 + Q_4$$

$$0.00884 = 3Q_4 + Q_4$$

$$\Rightarrow Q_4 = 0.00221 \text{ m}^3/\text{s}$$

$$Q_3 = 3Q_4 = 0.00663 \text{ m}^3/\text{s}$$

Also, for Section ③,

$$Q_3 = A_3 v_3$$

$$0.00663 = \frac{\pi}{4} \times D_3^2 \times 1.5$$

$$\Rightarrow D_3 = 75 \text{ mm}$$

For Section ④

$$Q_4 = A_4 v_4$$

$$0.00221 = \frac{\pi}{4} \times (30 \times 10^{-3})^2 \times v_4$$

$$\Rightarrow v_4 = 3.13 \text{ m/s}$$

⑥ Solⁿ:-

Flow physically possible or not?

Continuity equation satisfied $\Rightarrow$ possible
not satisfied $\Rightarrow$ not possible

For polar co-ordinate system,

Continuity eqⁿ

$u_z = 0$

$$\frac{\partial(u_r r)}{\partial r} + \frac{\partial(u_\theta)}{\partial \theta} = 0$$

or, $\frac{\partial(u_r r)}{\partial r} + \frac{\partial u_\theta}{\partial \theta} = 0$ - (1)

$$u_r = r \sin \theta$$

$$r u_r = r^2 \sin \theta$$

$$\frac{\partial(r u_r)}{\partial r} = 2r \sin \theta$$

$$u_\theta = 2r \cos \theta$$

$$\frac{\partial u_\theta}{\partial \theta} = -2r \sin \theta$$

Thus from (1)

$$2r \sin \theta - 2r \sin \theta = 0$$

satisfied

Physically possible flow.

⑦ Solⁿ:-

Sideslope

$$1:z$$

$$\frac{1}{z} = \tan 45^\circ = \text{slope}$$

$$\Rightarrow z = 1$$

Thus,

$$\text{Area} = b \times y + 2 \times \frac{1}{2} \times zy \cdot y$$

$$= by + zy^2$$

$$= 2 \times 2 + 1 \times 2^2$$

$$A_1 = 8 \text{ m}^2$$

⑧

For trap,

$$v_1 = 4 \text{ m/s}$$

For circular section (full)

$$v_2 = 2.5 \text{ m/s} \quad y = D$$

Depth of water (y) = Dia (D)

Thus,

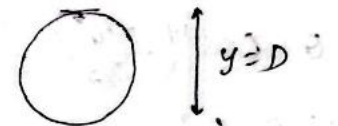
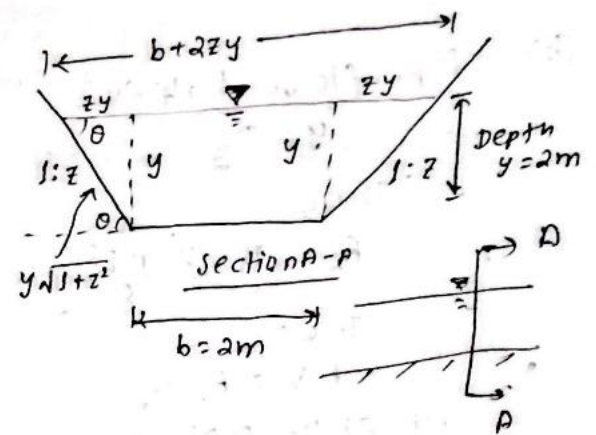
$$A_2 = \frac{\pi \times D^2}{4}$$

From continuity eqⁿ (since, same water goes from trap section to circular section)

$$A_1 v_1 = A_2 v_2$$

$$8 \times 4 = \frac{\pi \times D^2}{4} \times 2.5$$

$$\Rightarrow D = 4.04 \text{ m}$$



⑥ Pipe flows half full

$$y = D/2$$

Thur,

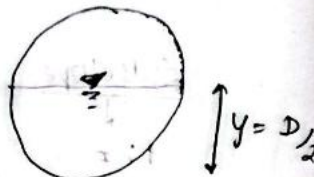
$$A_2 = \frac{\pi D^2}{8} \quad (\text{half of circular section})$$

Thur,

$$A_1 V_1 = A_2 V_2$$

$$\text{on } 8 \times 4 = \frac{\pi D^2}{8} \times 2.5$$

$$\Rightarrow D = 5.71 \text{ m}$$



⑦ Pipe flows at a depth of one half the radius

$$\theta = \tan^{-1} \left(\frac{D/4}{D/2} \right)$$

$$\theta = \cos^{-1} \left(\frac{D/4}{D/2} \right)$$

$$\Rightarrow \theta = 60^\circ$$

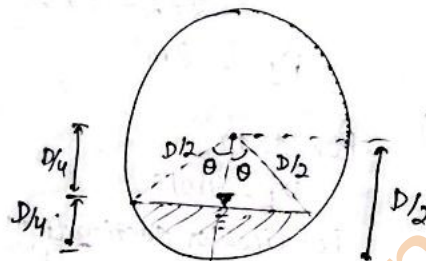
$$2\theta = 120^\circ$$

Thur,

$$A_2 = A_{\text{sector}} - 2A_{\Delta}$$

$$= \left(\frac{\pi D^2}{4} \right) \times \frac{120^\circ}{360^\circ} - 2 \times \frac{1}{2} \times \frac{D}{4} \times \sqrt{\left(\frac{D}{2} \right)^2 - \left(\frac{D}{4} \right)^2}$$

$$= 0.1535 D^2$$



Thur,

$$A_1 V_1 = A_2 V_2$$

$$\text{on } 8 \times 4 = 0.1535 D^2 \times 2.5$$

$$\Rightarrow D = 9.13 \text{ m}$$

⑧ Solⁿ.

Steady, incompressible 2-D velocity field.

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\vec{V} = (4.35 + 0.656x)\hat{i} + (-1.22 - 0.656y)\hat{j} \text{ m/s.}$$

Analytical expression for flow stream line $\rightarrow$?

$\times$ upper right quadrant: x & y positive.

$\times$ stagnation point: velocity zero at pt.
 $[u=0 \text{ \& } v=0, w=0]$

$\times$ Eqⁿ of stream line.

$$\frac{dy}{dx} = \frac{v}{u}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\int \frac{dx}{4.35 + 0.656x} = \int \frac{dy}{-1.22 - 0.656y}$$

$$\text{or, } \ln(4.35 + 0.656x) = -\ln(1.22 + 0.656y) + \ln C'$$

$$\text{or, } \ln[(4.35 + 0.656x) \times (1.22 + 0.656y)] = \ln C'$$

$$\text{on } 5.307 + 0.8x + 2.854y + 0.43xy = C'$$

$$\text{or, } x + 3.57y + 0.54xy = C' \quad \checkmark$$

For stagnation pt

$$u = 0$$

$$\text{on } 4.35 + 0.656x = 0$$

$$\text{on } x = -6.63$$

$$v = 0$$

$$-1.22 - 0.656y = 0$$

$$\text{on } y = -1.86$$

Stagnation pt.
(-6.63, -1.86)

Drawing several streamlines,
From eqⁿ of streamline,

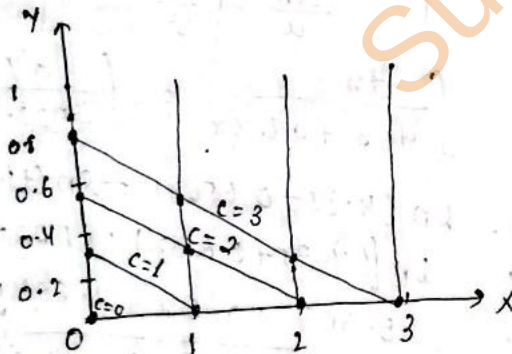
$$x + 0.57y + 0.54xy = c$$

$$x: 0 \rightarrow 3$$

$$y: 0 \rightarrow 3$$

$$y = \frac{c-x}{0.57+0.54x}$$

c = 0		c = 1		c = 2		c = 3	
x	y	x	y	x	y	x	y
0	0	0	0.28	0	0.56	0	0.84
1	-ve	1	0	1	0.24	1	0.49
2	-ve	2	-ve	2	0	2	0.2
3	-ve	3	-ve	3	-ve	3	0



Q Soln:-

$$u = a y \sin xy$$

$$v = a x \sin xy$$

velocity potential function = ?

$$\frac{\partial \phi}{\partial x} = -u = -a y \sin xy \quad (1)$$

$$\frac{\partial \phi}{\partial y} = -v = -a x \sin xy \quad (2)$$

Integrating (1)

$$\int \frac{\partial \phi}{\partial x} dx = \int -a y \sin xy dx$$

$$\phi = a y \frac{\cos xy}{y} + \frac{c}{y} \quad (3)$$

$\phi = a \cos xy + \frac{c}{y}$
c → constant independent of x may be function of y.

Diff. eqⁿ (3) w.r.t. y,

$$\frac{\partial \phi}{\partial y} = -a x \sin xy + f'(y) \quad (4)$$

Comparing Equating (3) & (4)
 $f'(y) = 0 \rightarrow f(y) = \frac{c}{y} \approx 0$
say 0.

Thus,

From (3), velocity potential function, $\phi = a \cos xy$

(10) → Solⁿ:

$$V = u\hat{i} + v\hat{j} + 0\hat{k}$$

$$V = (x+y)\hat{i} + (x^3-y)\hat{j}$$

$$u = x+y$$

$$v = x^3-y$$

⇒ Stream function exist or not
 ↳ If satisfies continuity equation
 ↓
 exist.

① Thus,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial (x+y)}{\partial x} + \frac{\partial (x^3-y)}{\partial y}$$

$$= 1 - 1$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \{\text{satisfied}\}$$

↓
 Stream function exists
 (flow occurs)

Yes.

② Stream function, $\psi = ?$

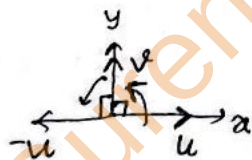
$$\frac{\partial \psi}{\partial x} = v = x^3 - y$$

$$\int \partial \psi = \int (x^3 - y) \partial x$$

$$\psi = \frac{x^4}{4} - xy + \frac{C}{2} \quad \text{--- ①}$$

----- may be $f(y)$

$$\frac{\partial \psi}{\partial y} = -u = -x - y \quad \text{--- ②}$$



Diff. ① ^{partially} w.r.t. y,

$$\frac{\partial \psi}{\partial y} = 0 - x + f'(y) \quad \text{--- ③}$$

Comparing ② & ③

$$-y = f'(y)$$

$$\text{or, } \underline{-\frac{y^2}{2} = f(y) = C} \quad \text{--- ④}$$

Thus, from ①,

$$\boxed{\psi = \frac{x^4}{4} - xy - \frac{y^2}{2}} \quad \text{--- ⑤}$$

③ Flow Irrotational or not :-

write rotational component about z
 Since, x & y given, find ω_z .

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} (3x^2 - 1) \neq 0$$

So, flow is rotational.

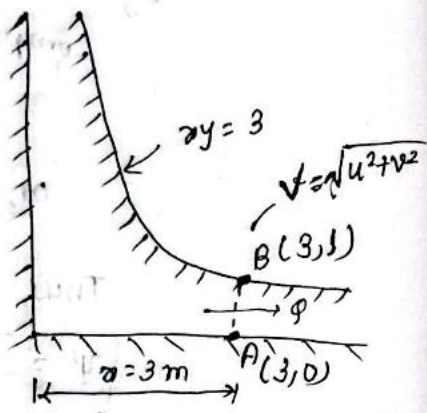
For irrotational, $\omega_z = 0$.

$$(11) \rightarrow \text{So } \psi:$$

$$\psi = 4xy \text{ cm}^2/\text{sec}$$

(x, y → m)

4 is dimensional constant & has unit $\frac{\text{cm}^2}{\text{s m}^2}$



$$a) \frac{\partial \psi}{\partial x} = v = 4y$$

$$\frac{\partial \psi}{\partial y} = -u = 4x$$

At B(3,1)

$$v = 4 \times 1 = 4$$

$$u = -\frac{4 \times 3}{100} = -\frac{12}{100} = -0.12$$

Thus, 4 is dimensional constant & has unit $\frac{\text{cm}^2}{\text{s m}^2}$

$$\text{So, } u_B = -\frac{4 \times 3}{100} \times 3 \text{ m} = -\frac{4 \times 3}{100} \times \frac{1}{100} \text{ cm/s}$$

$$= -0.12 \text{ cm/s}$$

$$v_B = \frac{4 \times 1}{100} \text{ cm/s} = 0.04 \text{ cm/s}$$

$$\text{So, } V_B = \sqrt{u^2 + v^2} = 0.1265 \text{ cm/s}$$

$$b) \text{ convective accel} \left(\frac{w \cdot r \cdot t}{t} \Rightarrow 0 \right) \cdot a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

At B

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$= (-0.12) \text{ cm/s} \times 0 + (0.04 \text{ cm/s}) \times 0$$

$$+ (0.04 \text{ cm/s}) \times 0$$

$$= 0.48 \text{ cm/s}^2 \times (1 \times 10^{-4})$$

$$4.8 \times 10^{-5} \text{ cm/s}^2$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$= (-0.12 \text{ cm/s}) \times 0 + (0.04 \text{ cm/s}) \times \frac{4 \text{ cm}^2}{\text{s m}^2}$$

$$= 0.04 \times 4 \times 1 \times 10^{-4} \text{ cm/s}^2$$

$$= 1.6 \times 10^{-5} \text{ cm/s}^2$$

$$\text{So, } a_{\text{convective}} = \sqrt{a_x^2 + a_y^2}$$

$$= 5.06 \times 10^{-5} \text{ cm/s}^2$$

c)

$$Q = A \times \text{velocity} \quad (\text{flow per unit width of } AB)$$

$$= A \times u$$

$$= (1 \times AB) \times u$$

$$= (1 \times 1) \times -0.12 \text{ cm/s}$$

$$= -0.12 \times 1 \times 10^4 \text{ cm}^3/\text{s}$$

$$= -1200 \text{ cm}^3/\text{s}$$

-ve sign indicates flow in -ve x-dir.

Alternatively

Two streamlines

↳ Along B.

↳ Along A.

Along B, $\psi_B = 4x \cdot \frac{x \times y}{3}$

$$= \frac{4 \text{ cm}^2}{5 \text{ m}^2} \times 3 \text{ m} \times 3 \text{ m}$$

$$= 4 \text{ cm}^2/\text{s}$$

Along A, $\psi_A = 4xy$

$$= 0 \text{ cm}^2/\text{s}$$

So,

flow, $Q = (\psi_B - \psi_A) \times \frac{\text{width}}{\text{per unit width}}$

$$= (4 - 0) \times 1 \text{ m}$$

$$= 4 \text{ cm}^2/\text{s} \times 100 \text{ cm}$$

flow rate = 400 cm²/s

(2) Solⁿ:

$$u = \frac{(y^2 - x^2)}{(x^2 + y^2)^2}$$

$$v = \frac{-2xy}{(x^2 + y^2)^2} \quad (2 \text{ D flow})$$

Fluid is incompressible,
steady, incompressible continuity eqⁿ satisfied
i.e. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ } steady, incompressible flow.

$$\frac{\partial u}{\partial x} = \frac{\{(x^2 + y^2)^2 - 2x\} - \{(y^2 - x^2) \cdot 2 \cdot 2x(x^2 + y^2)\}}{(x^2 + y^2)^4}$$

$$= \frac{-2x(x^2 + y^2)^2 + [4x(x^2 + y^2)(x^2 - y^2)]}{(x^2 + y^2)^4}$$

$$= \frac{-2x(x^2 + y^2)[x^2 + y^2 - 2(x^2 - y^2)]}{(x^2 + y^2)^4}$$

$$= \frac{-2x[x^2 + y^2 - 2x^2 + 2y^2]}{(x^2 + y^2)^3}$$

$$= \frac{-2x[3y^2 - x^2]}{(x^2 + y^2)^3}$$

$$\frac{\partial v}{\partial y} = \frac{-2x(x^2 + y^2)^2 + 2xy \cdot 2 \cdot 2y \cdot (x^2 + y^2)}{(x^2 + y^2)^4}$$

$$= \frac{-2x(x^2 + y^2)[x^2 + y^2 - 4xy^2]}{(x^2 + y^2)^4}$$

$$= \frac{+2x[3y^2 - x^2]}{(x^2 + y^2)^3}$$

Thus, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, So flow is incompressible.

⑤ Flow is irrotational, $\omega_z = ?$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \quad \text{--- (1)}$$

$$\begin{aligned} \frac{\partial v}{\partial x} &= \frac{\partial}{\partial x} \left(\frac{-2xy}{(x^2+y^2)^2} \right) \\ &= \frac{-2y(x^2+y^2)^2 + 2xy \cdot 2 \cdot 2x \cdot (x^2+y^2)}{(x^2+y^2)^4} \\ &= \frac{-2y(x^2+y^2) [x^2+y^2 - 4x^2]}{(x^2+y^2)^4} \\ &= \frac{-2y [y^2 - 3x^2]}{(x^2+y^2)^3} = \frac{2y (3x^2 - y^2)}{(x^2+y^2)^3} \end{aligned}$$

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y} \left(\frac{y^2 - x^2}{(x^2+y^2)^2} \right) \\ &= \frac{2y(x^2+y^2)^2 - (y^2 - x^2) \cdot 2 \cdot 2y \cdot (x^2+y^2)}{(x^2+y^2)^4} \\ &= \frac{2y [x^2+y^2 - 2y^2 + 2x^2]}{(x^2+y^2)^3} \\ &= \frac{2y (3x^2 - y^2)}{(x^2+y^2)^3} \end{aligned}$$

Thus, $\omega_z = 0$. So, flow is irrotational.

⑥ Streamline equation / function, $\psi = ?$

$$\frac{\partial \psi}{\partial x} = v = \frac{-2xy}{(x^2+y^2)^2} \quad \text{--- (1)}$$

$$\frac{\partial \psi}{\partial y} = -u = \frac{y^2 - x^2}{(x^2+y^2)^2} \quad \text{--- (11)}$$

Integrating (1)

$$\int \partial \psi = - \int \frac{2xy}{(x^2+y^2)^2} \partial x$$

$$\psi = -y \int (x^2+y^2)^{-2} \cdot \frac{2x \partial x}{2}$$

$$\psi = -y \frac{(x^2+y^2)^{-2+1}}{-2+1} + C$$

$$\psi = \frac{y}{(x^2+y^2)} + f(y) \quad \text{--- (3)}$$

Diff. w.r.t. y ,

$$\frac{\partial \psi}{\partial y} = \frac{(x^2+y^2) - y \cdot 2y}{(x^2+y^2)^2} + f'(y)$$

$$\frac{\partial \psi}{\partial y} = \frac{x^2 - y^2}{(x^2+y^2)^2} + f'(y) \quad \text{--- (4)}$$

Comparing (11) & (4)

$$f'(y) = 0$$

$$\therefore f(y) = C, \text{ say } C(0) \rightarrow \text{can be taken as zero(0)}$$

Then,

eqⁿ of streamline,

$$\psi = \frac{y}{x^2 + y^2} \quad \text{--- (5)}$$

When, putting (2, 2)

$$\psi = \frac{2}{4+4} = \frac{1}{4}$$

Putting (1, 2-√3)

$$\begin{aligned} \psi &= \frac{2-\sqrt{3}}{1+4-2\sqrt{3}+3} \\ &= \frac{2-\sqrt{3}}{4(2-\sqrt{3})} \\ &= \frac{1}{4} \end{aligned}$$

Since, value of ψ is same at both points (2, 2) & (1, 2-√3) lies on same streamline

② At (1, 1), $\psi_1 = \frac{1}{1+1} = \frac{1}{2}$

At, (2, 2) $\psi_2 = \frac{2}{4+4} = \frac{1}{4}$

Then,

Discharge, $Q = (\psi_1 - \psi_2) \times \text{width}$
 $= \left(\frac{1}{2} - \frac{1}{4}\right) \times 1$

$$Q = \frac{1}{4} \text{ u}$$

(13) Solⁿ:-

stream function, $\psi = 2xy$

$$0 \leq (x, y) \leq 5$$

① At (1, 2),

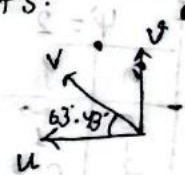
$$\frac{\partial \psi}{\partial x} = v = 2y = 4 \text{ units}$$

$$\begin{aligned} \frac{\partial \psi}{\partial y} &= -u = 2x \\ \Rightarrow u &= -2x = -2 \text{ units} \end{aligned}$$

Thus, $V = -2\vec{i} + 4\vec{j}$ units

$V = 4.47$ units

$\theta = \tan^{-1}\left(\frac{4}{-2}\right) = -63.43^\circ$



Now,

$$-v = \frac{\partial \phi}{\partial y} = -2y$$

or, $\int \partial \phi = \int 2y \partial y$
 $\phi = -y^2 + C$ --- (2)

$$-u = \frac{\partial \phi}{\partial x} = +2x$$

Comparing,
 Diff. ② $\frac{\partial \phi}{\partial x} = f'(x) \Rightarrow f(x) = +x^2$

Thus,

$\phi = x^2 - y^2$ → velocity function

(b) Stream lines, ψ const. line. $\psi = 2xy$
 $\psi(1,1) = 2 \times 1 \times 1 = 2$
 $\psi(1,2) = 2 \times 1 \times 2 = 4$
 $\psi(2,2) = 2 \times 2 \times 2 = 8$ } passing thr. these
 Thus, $y = \frac{\psi}{2x}$

$xy = 0.5$
 $0 \rightarrow 5$

$\psi = 2$		$\psi = 4$		$\psi = 8$	
x	y	x	y	x	y
0	∞	0	∞	0	∞
1	1	1	2	1	4
2	0.5	2	1	2	2
3	0.33	3	0.66	3	1.33
4	0.25	4	0.5	4	1
5	0.2	5	0.4	5	0.8

Equipotential line (ϕ const line)
 ϕ can be,

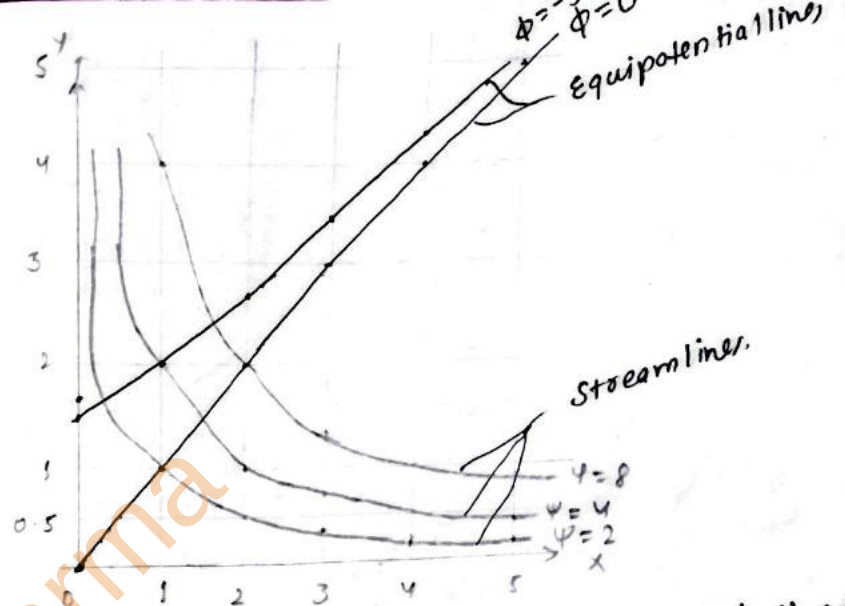
$$\phi(1,1) = 0$$

$$\phi(1,2) = -3$$

$$\phi(2,2) = 0$$

$$y = \sqrt{x^2 - \phi}$$

$\phi = 0$		$\phi = -3$	
x	y	x	y
0	0	0	1.73
1	1	1	2
2	2	2	2.65
3	3	3	3.46
4	4	4	4.36
5	5	5	5.29



(c) Stream lines (ψ const)
 For $\psi = 100$ For $\psi = 300$
 $2xy = 100$ $2xy = 300$
 $y = \frac{50}{x}$ $y = \frac{150}{x}$

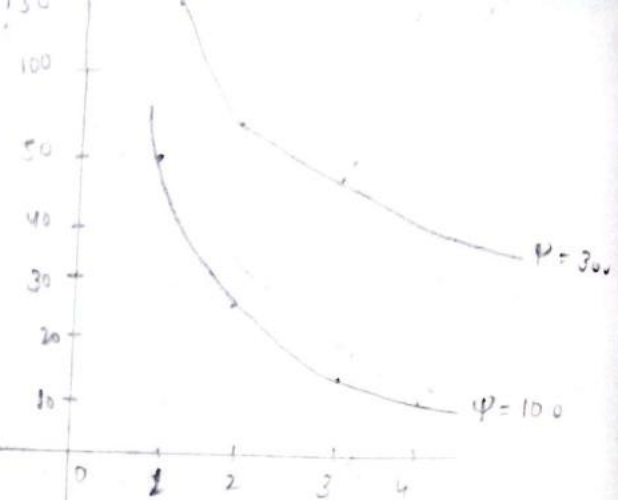
Range not given.

$\psi = 100$		$\psi = 300$	
x	y	x	y
0	∞	0	∞
1	50	1	150
2	25	2	75
3	16.66	3	50
4	12.5	4	37.5
-1	-50	-1	-150
-2	-25	-2	-75
-3	-16.66	-3	-50

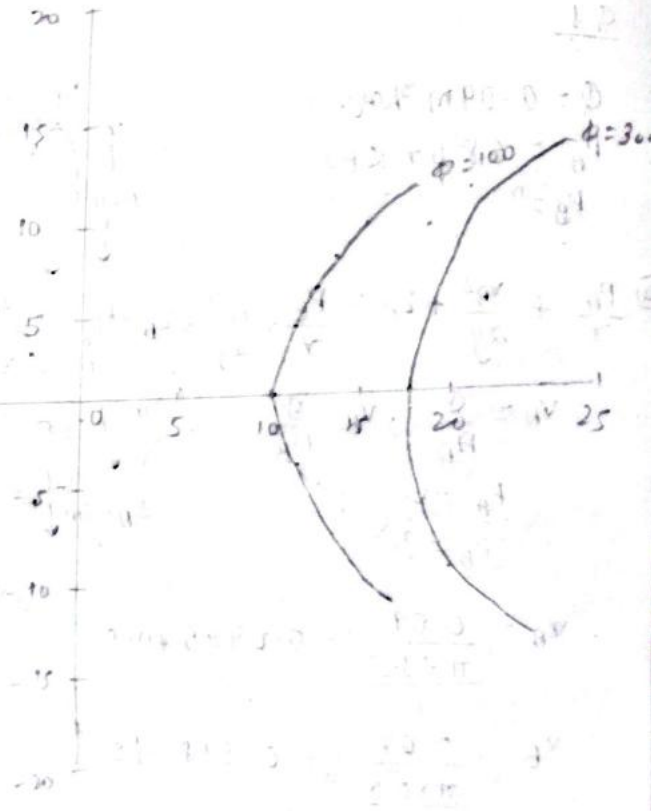
Equipotential line (ϕ const)
 For $\phi = 100$ For $\phi = 300$
 $x^2 - y^2 = 100$ $x^2 - y^2 = 300$
 $\Rightarrow y = \sqrt{x^2 - 100}$ $y = \sqrt{x^2 - 300}$
 $(x \geq 10)$ $(x \geq 17.32)$

$\phi = 100$		$\phi = 300$	
x	y	x	y
10	0	20	± 10
11	± 4.58	21	± 11.87
12	± 6.6	22	± 13.56
13	± 6.83	23	± 15.13
14	± 6.98	24	± 16.6
-11	± 4.6	-21	± 11.87
-12	± 6.6	-22	± 13.56
-13	± 6.83	-23	± 15.13
-14	± 6.98	-24	± 16.6

Streamlines



Equipotential line



Surendra Sharma

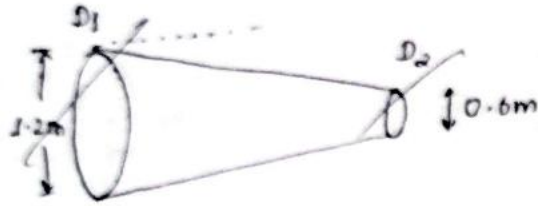
chapter-8 Equation of motion & energy eq²

Q.1

$$Q = 0.09 \text{ m}^3/\text{sec}$$

$$P_A = 68.67 \text{ KPa}$$

$$P_B = ?$$



$$\textcircled{a} \frac{P_A}{\gamma} + \frac{v_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{v_B^2}{2g} + z_B + h_L$$

$$v_A = \frac{Q}{A_A}, v_B = \frac{Q}{A_B}$$

$$P_A = ?$$

$$P_B = ?$$

$$v_A = \frac{0.09}{\pi \times 1.2^2} = 0.0796 \text{ m/s}$$

$$v_B = \frac{0.09}{\pi \times 0.6^2} = 0.318 \text{ m/s}$$

Thus,

$$\frac{68.67}{9.81} + \frac{(0.0796)^2}{2 \times 9.81} + 3 = \frac{P_B}{9.81} + \frac{(0.318)^2}{2 \times 9.81} + 0$$

$$P_B = 98.053 \text{ KPa}$$

$$\textcircled{b} \text{ If } P_A = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$$

$$P_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$$

Let, head loss be at B side i.e. $(h_L)_B$

$$\text{Using Bernoulli, } \frac{P_A}{\gamma} + \frac{v_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{v_B^2}{2g} + z_B + (h_L)_B$$

$$(h_L)_B = \left(\frac{9.81 \times 10^4}{9.81 \times 1000} + \frac{0.0796^2}{2 \times 9.81} + 3 \right) - \left(\frac{5.886 \times 10^4}{9.81 \times 1000} + \frac{0.318^2}{2 \times 9.81} + 0 \right)$$

$$(h_L)_B = 7 \text{ m} \quad +ve,$$

So, Liquid flows from A to B.

$E_A > E_B$
Liquid flows from higher energy to lower energy.

(2) Solⁿ: $\alpha = ?$ (K-E correction factor)

$$\alpha = 10 \left(1 - \left(\frac{r}{R} \right)^2 \right) \quad (\text{Pipe})$$

$$\text{radius of pipe} = 1 \text{ cm} = 0.01 \text{ m}$$

We know,

$$\alpha = \frac{\int v^3 dA}{v^3 A} \quad \text{--- (1)}$$

$\{ v = \text{avg. velocity} \}$

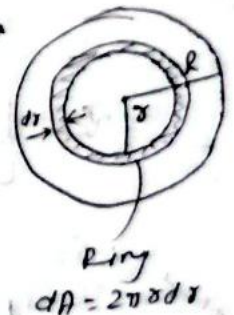
$$Q = AV \quad \text{continuity eq.}$$

$$\int dQ = \int dAV \quad [\because Q = \int dQ]$$

$$V = \frac{dQ}{dA} = \frac{\int v dA}{A}$$

$$\text{Thus, } V = \frac{\int v dA}{A} = \frac{\int_0^{0.01} 10 \left(1 - \left(\frac{r}{0.01} \right)^2 \right) \cdot 2\pi r \cdot dr}{\pi \times 0.01^2}$$

$$V = 5 \text{ m/sec.}$$



chapter 8-8 Equation of motion & energy eq².

Q.1

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$$P_A = 68.67 \text{ KPa}$$

$$P_B = ?$$

$$\textcircled{a} \frac{P_A}{\gamma} + \frac{v_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{v_B^2}{2g} + Z_B + h_L$$

$$v_A = \frac{Q}{A_A}, v_B = \frac{Q}{A_B}$$

$$P_A = ?$$

$$v_A = \frac{0.09}{\frac{\pi \times 1.2^2}{4}} = 0.0796 \text{ m/s}$$

$$v_B = \frac{0.09}{\frac{\pi \times 0.6^2}{4}} = 0.318 \text{ m/s}$$

Thus,

$$\frac{68.67}{9.81} + \frac{(0.0796)^2}{2 \times 9.81} + 3 = \frac{P_B}{9.81} + \frac{(0.318)^2}{2 \times 9.81} + 0$$

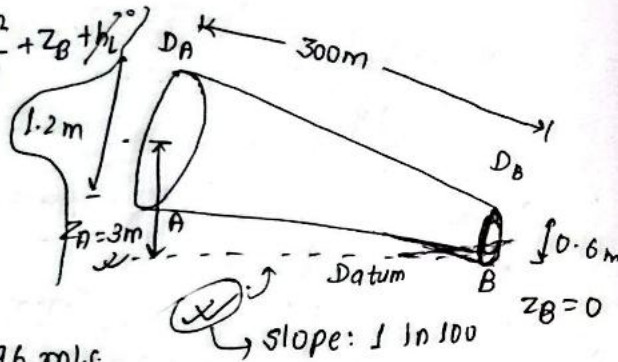
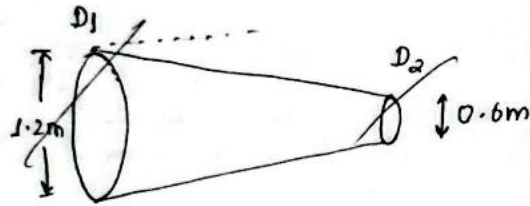
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$$(h_L)_B = \left(\frac{9.81 \times 10^4}{9.81 \times 1000} + \frac{0.0796^2}{2 \times 9.81} + 3 \right) - \left(\frac{5.886 \times 10^4}{9.81 \times 1000} + \frac{0.318^2}{2 \times 9.81} + 0 \right)$$

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So, Liquid flows from A to B.

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$\{ v = \text{avg. velocity} \}$

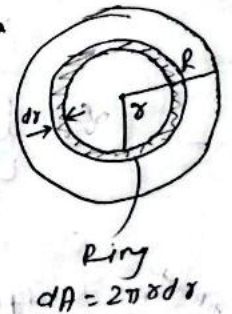
$$\{ Q = AV \} \text{ continuity eq.}$$

$$\{ Q = \int u dA \} \quad [\because Q = \int u dA]$$

$$v = \frac{\int u dA}{A}$$

$$\text{Thus, } v = \frac{\int u dA}{A} = \frac{\int_0^{0.01} 10 \left(1 - \left(\frac{r}{0.01} \right)^2 \right) \cdot 2\pi r \cdot dr}{\pi \times 0.01^2}$$

$$v = 5 \text{ m/sec.}$$



Thus,

$$\alpha = \frac{\int v^3 dA}{V^3 A}$$

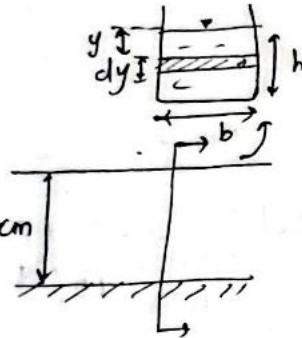
$$= \frac{\int_{y=0}^{0.01} \left[10 \left(1 - \frac{y^2}{(0.01)^2} \right) \right]^3 2\pi y dy}{5^3 \times \pi \times 0.01^2}$$

$$= \frac{0.0785}{5^3 \times \pi \times 0.01^2}$$

$$= 2$$

⑥ $u = 10 \left(1 - \left(\frac{y}{h} \right)^2 \right)$

2cm deep channel
 $h = 2\text{cm} = 0.02\text{m}$



$$\alpha = \frac{\int u^3 dA}{U^3 A}$$

breadth of channel not given
 got cancelled.

continuity eqⁿ:

$$Q = AU \text{ } \rightarrow \text{ overall}$$

$$\int dQ = \int u dA \text{ } \rightarrow \text{ small strip}$$

$$dA = dy b$$

$$\int dQ = \int u b dy$$

$$Q = \int dQ$$

$$AU = \int u b dy$$

$$h b U = \int u b dy$$

$$\Rightarrow U = \frac{\int u dy}{h} \text{ } \rightarrow \text{ avg. velocity}$$

$$U = \frac{\int_{y=0}^{0.02} 10 \left(1 - \left(\frac{y}{0.02} \right)^2 \right) dy}{0.02}$$

$$U = 6.67 \text{ m/s}$$

Thus,

$$\alpha = \frac{\int_{y=0}^{0.02} \left[10 \left(1 - \left(\frac{y}{0.02} \right)^2 \right) \right]^3 \frac{b dy}{6.67^3 \times b \times h_{0.02}}}{6.67^3 \times b \times h_{0.02}}$$

$$= \frac{9.14}{6.67^3 \times 0.02}$$

$$= 1.54$$

③ Solⁿ:

convert all fluids into
 a same one
 Here, say oil.

For gasoline.

$$p_{\text{gas}} = 0.68 \times 9.81 \times 2 = 0.8 \times 9.81 \times h_g$$

$$\text{oil} \Rightarrow h_g = 1.7 \text{ m}$$

$$p_{\text{air}} = 120 \text{ kPa}$$

$$\text{oil} \Rightarrow h_{\text{air}} = \frac{120}{0.8 \times 9.81} = 15.29 \text{ m}$$

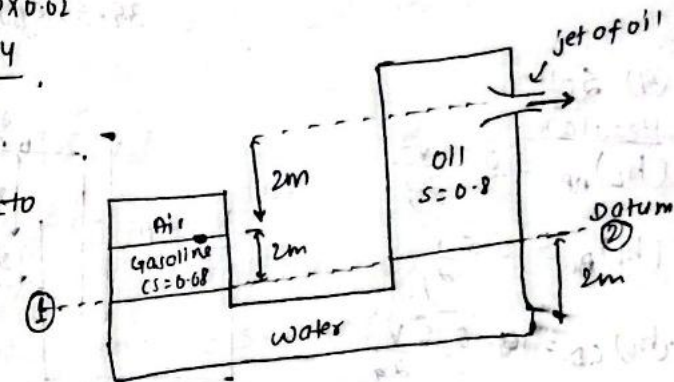
Energy eqⁿ:

$$E_1 = E_2 + \text{loss} \text{ } [\text{Interms of oil}]$$

$$[z_1 = 0, \text{ datum }]$$

$$\text{or, } \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

$$\text{or, } (1.7 + 15.29) + 0 + 0 = 0 + \frac{v_2^2}{2g} + 4$$



$$v_2 = 15.96 \text{ m/s}$$

$$\text{Mass flow rate, } \frac{dm}{dt} = ?$$

$$Q = A v_2 \text{ } \{ \text{jet} \}$$

$$= \frac{\pi \times (20 \times 10^{-3})^2}{4} \times 15.96$$

$$= 5.02 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\frac{dm}{dt} = Q \times \rho = 5.02 \times 10^{-3} \times 0.8 \times 9810 \times 1000$$

$$= 39.95 \text{ kg/s} = 4.011 \text{ kg/s}$$

$$s = \frac{m}{v}$$

$$m = v \times s$$

(4) Solⁿ

Head loss,

$$(h_L)_{AB} = 0.3 \frac{v^2}{2g}$$

$$(h_L)_{BC} = 0.5 \frac{v^2}{2g}$$

$$(h_L)_{CD} = 0.5 \frac{v^2}{2g}$$

Velocity head.

$$Q = 3 \text{ m}^3/\text{sec. } \{ \text{thr. pipe} \}$$

$$Q = A v$$

$$A = \frac{\pi \times (600 \times 10^{-3})^2}{4}$$

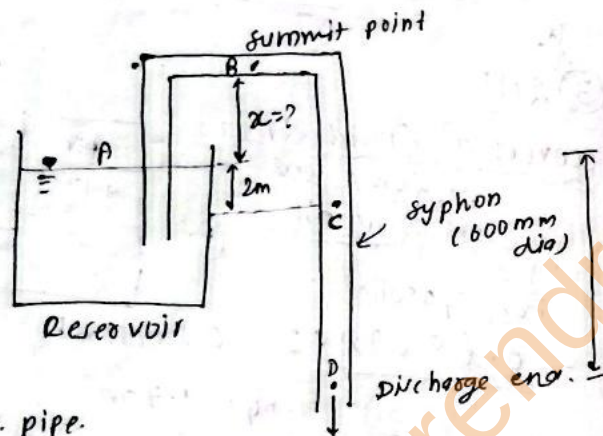
$$\Rightarrow v = \frac{Q}{A} = 10.61 \text{ m/s}$$

Then,

$$(h_L)_{AB} = 1.72 \text{ m}$$

$$(h_L)_{BC} = 2.87 \text{ m}$$

$$(h_L)_{CD} = 2.87 \text{ m}$$



Summit point: max^m elevation of pipe s.t. syphon action take place.

From A to B

$$E_A = E_B + (h_L)_{AB}$$

Say, level of A as datum,

Datum can be taken anywhere
Datum: $\uparrow +ve$
 $\downarrow -ve$

$$\frac{P_A}{\gamma} + \frac{v_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{v_B^2}{2g} + Z_B + (h_L)_{AB}$$

or, $\frac{P_{atm}}{\gamma} + 0 + 0 = \frac{P_{atm}}{\gamma} + \frac{v_B^2}{2g} + x + 1.72$

$$\frac{1 \times 10^5}{9.81 \times 1000} + 0 + 0 = \frac{20 \times 10^3}{9810} + \frac{10.61^2}{2 \times 9.81} + x + 1.72$$

$$\Rightarrow x = 0.697 \text{ m}$$

$\{ (h_L)_{AC} = (h_L)_{AB} + (h_L)_{BC} \}$

(b)

Energy eqⁿ:-

{ considering level of A as datum }

$$E_B = E_C + (h_L)_{BC}$$

$$\text{or, } \frac{20 \times 10^3}{9810} + \frac{10.61^2}{2 \times 9.81} + 0.697 = (h_p)_C + \frac{10.61^2}{2 \times 9.81} - 2 + 2.87$$

$$\Rightarrow (h_p)_C = 1.866 \text{ m}$$

(c)

Energy eqⁿ:-

$E_C = E_D + (h_L)_{CD}$ { considering level of A, (or reservoir) as datum }

$$\text{or, } 1.866 + \frac{v^2}{2g} + (-2) = \frac{10^5}{9810} + \frac{v^2}{2g} - h' + 2.87$$

$$\Rightarrow h' = 13.2 \text{ m, } \{ \text{from reservoir surface} \}$$

'OP' Energy eqⁿ from A to D

$$E_P = E_D + (h_L)_{AD}$$

$$(h_L)_{AD} = (h_L)_{AB} + (h_L)_{BC} + (h_L)_{CD}$$

$$= 7.46 \text{ m}$$

Considering Gauge pressure,

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D + (h_L)_{AD}$$

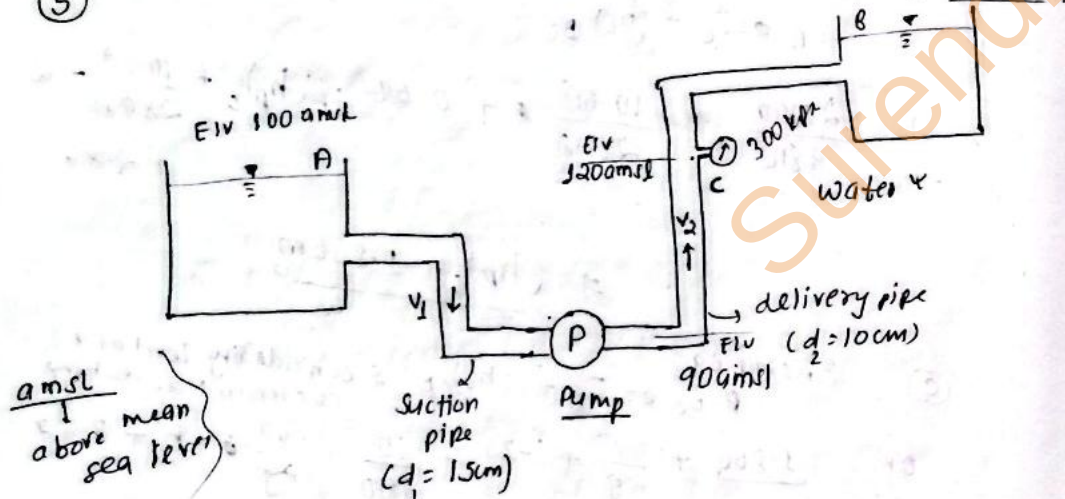
$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
 $P_{atm}=0$ static datum 0 (Datum) (Discharging)

$$0 + 0 + 0 = \frac{10.61^2}{2 \times 9.81} + Z_D + 7.46$$

$$\Rightarrow Z_D = -13.20 \text{ m}$$

-ve sign indicates Z_D is below considered datum.

⑤



$$(h_L)_{AP} = 1.5 \frac{V_1^2}{2g}$$

Vel. head of suction pipe

$$(h_L)_{PC} = 2 \frac{V_2^2}{2g}$$

Vel. head of delivery pipe.

$$(h_L)_{CB} = 4 \frac{V_2^2}{2g}$$

Q=?

Power supply=?

Energy eqⁿ जहाँ वॉट जहाँ सगन लगाउका नि हुन्छ
तत्र सजिलो जरी Anu आउनु पर्छो

Eg: A to B $V_A=0, P_A=0, P_B=0, V_B=0$
 $Z_A=100, Z_B=145$ $\frac{hp}{\gamma}$ $(h_L)_{total}$
 no info } NOUKE.

So, From C to B,

$$E_C = E_B + (h_L)_{CB}$$

$$\frac{300}{9.81} + \frac{V_2^2}{2g} + 120 = 0 + 0 + 145 + (h_L)_{CB}$$

$$4 \times \frac{V_2^2}{2 \times 9.81}$$

$$\Rightarrow \frac{3 V_2^2}{2 \times 9.81} = 5.581$$

$$\Rightarrow V_2 = 6.041 \text{ m/s}$$

So,

$$(h_L)_{PC} = 2 \times \frac{6.041^2}{2 \times 9.81} = 3.72 \text{ m}$$

$$(h_L)_{CB} = 7.44 \text{ m.}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{\pi \times 0.1^2}{4} \times 6.041$$

$$\frac{\pi \times 0.15^2}{4}$$

$$V_1 = 2.685 \text{ m/s}$$

$$(h_L)_{Ap} = 0.55 \text{ m}$$

$$(h_L)_{Ap} = 1.5 \times \frac{2.685^2}{2 \times 9.81}$$

$$= 0.55 \text{ m}$$

Pump: does work on fluid (+)

So, head of pump, $h_p \rightarrow +v$

Now,

Energy eqⁿ from A to B,

$$E_p = E_b + (h_L)_{AB}$$

$$\text{Or, } 0 + 0 + 100 = 0 + 0 + 145 + (0.55 + 7.44 + 3.72)$$

$$+ h_p$$

$$\Rightarrow h_p = 56.71 \text{ m.}$$

$$Q = A_1 V_1 = 0.047 \text{ m}^3/\text{s}$$

$$\text{Power supplied (P)} = \frac{\gamma Q h_p}{\gamma_w}$$

$$= 26.143 \text{ kW.}$$

(b)

Just before (P)

$$E_p = E_p + (h_L)_{Ap}$$

∴ Pump सँग पुग्छा छैन 20, no hp)

$$\text{Or, } 0 + 0 + 100 = \frac{(P_p)_b}{9810} + \frac{V_1^2}{2g}$$

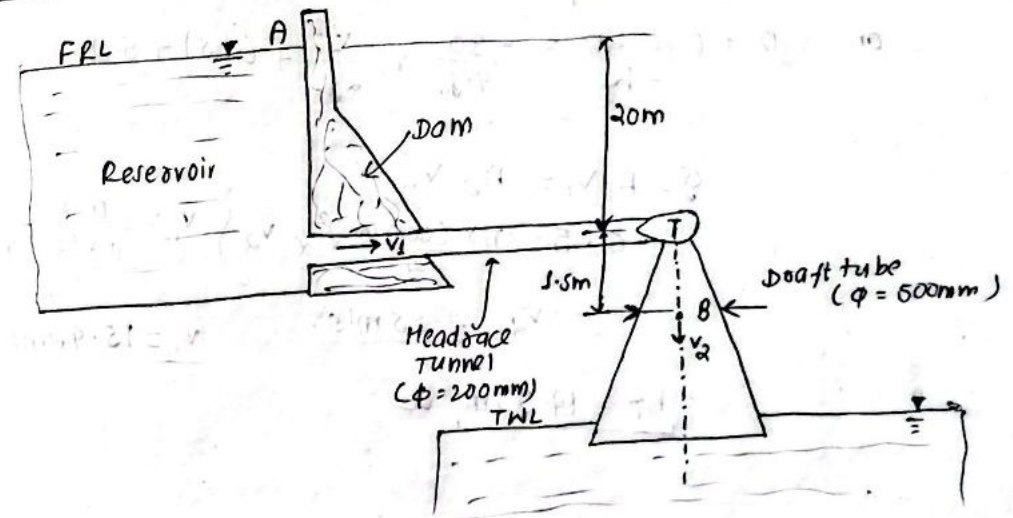
$$+ 90 + 0.55$$

$$(P_p)_{\text{before}} = 89.1 \text{ kPa}$$

(b)

Before, suction pipe को velocity i.e. V_1 लिने

(b) Solⁿ:



Just After (P)

Pump को head पनि राख्ने (h_p)

Just after $\Rightarrow$ pump कट्छो

$$E_p + h_p = E_p + (h_L)_{Ap}$$

$$0 + 0 + 100 + 56.71 = \frac{(P_p)_A}{9810} + \frac{V_2^2}{2g} + 90 + 0.55$$

$$\Rightarrow (P_p)_{\text{After}} = 645.43 \text{ kPa}$$

$$(P_p)_{\text{After}} = 630.78 \text{ kPa}$$

After, मा Delivery tube/pipeline को velocity i.e. V_2 लिने

$$(h_L)_{AT} = 2.5 \text{ m (of water)}$$

$$(h_L)_{TB} = 3 \text{ m (of oil)}$$

$$= \frac{3 \times 0.9 \times 9.81}{9.81}$$

$$= 2.7 \text{ m}$$

$$(h_L)_{TB} = 2.7 \text{ m (of water)}$$

$$P_B = -30 \text{ KPa}$$

Power generated by Turbine (P_T) = $\eta \gamma Q h_T$

0.85 0.5

(a) Energy equation from A to B,

$$E_A = E_B + (h_L)_{AB} \quad \left\{ \begin{array}{l} \text{Datum considered at} \\ \text{turbine level} \end{array} \right.$$

$$\text{or, } \frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + (2.5 + 2.7)$$

$$\text{or, } 0 + 0 + 20 = \frac{-30}{9.81} + \frac{V_2^2}{2g} + (7.5) + 5.2$$

$$Q = A_1 V_1 = A_2 V_2$$

$$0.5 = \frac{\pi \times (500 \times 10^{-3})^2}{4} \times V_2 \quad \left\{ \begin{array}{l} V_1 = \frac{0.5}{\frac{\pi \times (200 \times 10^{-3})^2}{4}} \\ V_1 = 15.92 \text{ m/s} \end{array} \right.$$

$$\Rightarrow V_2 = 2.55 \text{ m/s}$$

$$h_T = 19.03 \text{ m, } \checkmark$$

Thus,

$$P_T = 0.85 \times 9810 \times 0.5 \times 19.03$$

$$= 79.34 \text{ KW}$$

(b)

Just before turbine

$$E_A = E_T + (h_L)_{AT}$$

$$\text{or } 0 + 0 + 20 = \left(\frac{P_T}{\gamma} \right) + \frac{V_1^2}{2g} + 0 + 2.5$$

$$\left(\frac{P_T}{\gamma} \right)_{\text{before}} = 4.58 \text{ m}$$

Just after turbine

$$E_A - h_T = E_T + (h_L)_{AT}$$

$$0 + 0 + 20 - 19.03 = \left(\frac{P_T}{\gamma} \right)_A + \frac{V_2^2}{2g} + 0 + 2.5$$

$$\left(\frac{P_T}{\gamma} \right)_{\text{After}} = -1.86 \text{ m}$$

7 Solⁿ:

$D_1 = 50\text{mm}$
 $D_2 = 25\text{mm}$

flowing from ② to ①
 $P_2 - P_1 = 31\text{KPa}$

A-T-Q

$h_L = KV^2$ (head loss varies square of velocity)
 $K = ?$

Energy eqⁿ:

$E_2 = E_1 + h_L$ { head loss varies at every dia so, take small strip }
 section ③

For section, ③,

$dh_L = K V^2$
 $h_L = \int_0^2 K V^2 dx$

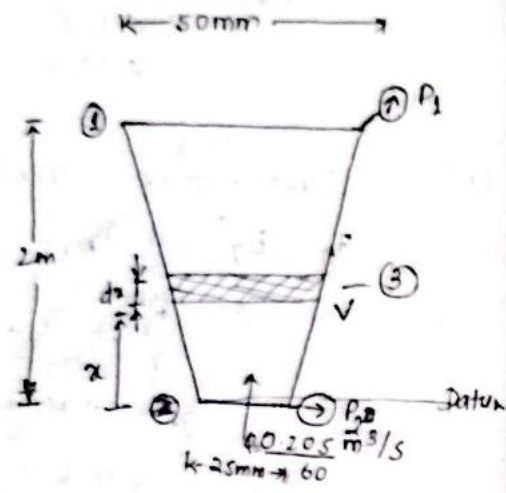
So,

$E_2 = E_1 + \int_0^2 K V^2 dx$

or, $\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + \int_0^2 K V^2 dx$

or, $\frac{P_2 - P_1}{\gamma} + \frac{Z_2 - Z_1}{2} + \frac{V_2^2 - V_1^2}{2g} = \int_0^2 K V^2 dx$
 $\frac{31}{9.81} + (-2) + \frac{6.96^2 - 1.74^2}{2 \times 9.81} = \int_0^2 K V^2 dx = 6.96 \text{ m/s}$

$V_1 = \frac{Q}{A_1} = \frac{0.205 \times 4}{60 \times \pi \times 50 \times 10^{-3}} = 1.74 \text{ m/s}$
 $V_2 = \frac{Q}{A_2} = \frac{0.205 \times 4}{60 \times \pi \times 25 \times 10^{-3}} = 6.96 \text{ m/s}$



$\int_0^2 K V^2 dx = 3.475 \text{ m}$

$V = \frac{Q}{A}$

(d) section 3 = ?

$d = 0.025 \left(\frac{\pi + 2}{2} \right)$ (linearly varies)

$V = \frac{0.205 \times 4}{60 \times \pi \left(0.025 \left(\frac{\pi + 2}{2} \right) \right)^2}$

$\int_0^2 K \left(\frac{27.84^2}{(\pi + 2)^2} \right) dx = 3.475$

$K = \frac{3.475}{28.26}$

$K = 0.123$

Next part,

If,

$P_1 - P_2 = 0 \text{ KPa}$
 & water is flowing downward
 $Q = ?$

still water may flow due to other energy.

Energy eqⁿ:

$E_1 = E_2 + h_L$

or, $\frac{P_1 - P_2}{\gamma} + 2 + \frac{V_1^2 - V_2^2}{2g} = 3.475 = 0$

or,

from ① & ②

Also, from continuity eqⁿ
 $A_1 V_1 = A_2 V_2$

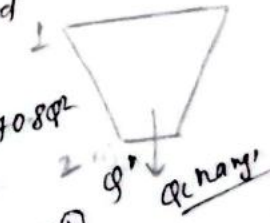
or, $50^2 \times V_1 = 25^2 \times V_2$
 $V_2 = 4 V_1$ — (1)

$\frac{V_1^2 - 16 V_1^2}{2g} + 2 = 3.475$

$= 297770.8 \left(\frac{\pi \times 0.05^4}{4} \times V_1^4 \right)$ from ① & ②

$V_1 = 1.0226 \text{ m/s}$

$Q = 3.34 \text{ m}^3/\text{s}$

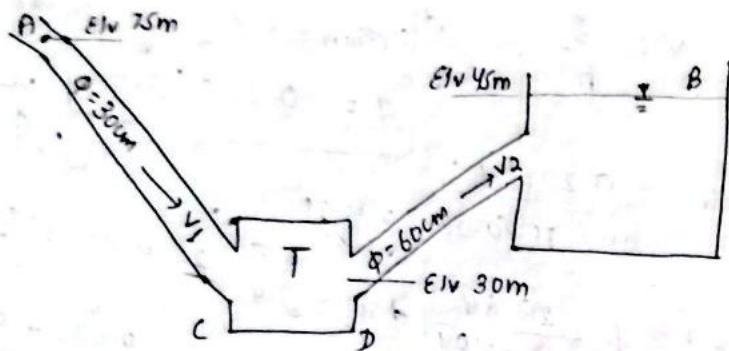


$h_L = \int_0^2 K V^2 dx$
 $= \int_0^2 \frac{K Q'^2}{\frac{\pi^2 \times 0.05^4}{4} \left(\frac{\pi + 2}{2} \right)^2} dx$
 $h_L = 297770.8 Q'^2$

Thus,

$Q' = A_1 V_1$
 $Q = 0.002 \text{ m}^3/\text{s}$

⑧ Soln:-



Energy equation:-

$$E_A - h_T = E_B + (h_L)_{AB}$$

$$\text{or, } \frac{P_A}{\gamma} + \frac{v_1^2}{2g} + 75 - 60 = \frac{P_B}{\gamma} + \frac{v_B^2}{2g} + 45$$

$$\left(\frac{3v_1^2}{2g} + \frac{2v_2^2}{2g} \right)$$

$$\text{or, } \frac{505}{9.81} + \frac{v_1^2}{2g} + 75 - 60 = 0 + 0 + \frac{3v_1^2}{2g} + \frac{2v_2^2}{2g}$$

$$\text{or, } 2v_1^2 + 2v_2^2 = 1304.34214$$

$$v_1^2 + v_2^2 = 652.171 - (1)$$

continuity eqn:-

$$Q = A_1 v_1 = A_2 v_2$$

$$\frac{\pi \times 30^2}{4} \times v_1 = \frac{\pi \times 60^2}{4} \times v_2$$

$$\Rightarrow v_1 = 4v_2 - (II)$$

From (I) & (II),

$$v_2 = 6.49 \text{ m/s } 3.52 \text{ m/s}$$

$$v_1 = 24.77 \text{ m/s } 14.08 \text{ m/s}$$

$$(a) \cdot Q = A_1 v_1 = \frac{\pi \times (30 \times 10^{-2})^2}{4} \times 14.08 = 0.995 \text{ m}^3/\text{sec}$$

$$(b) \text{ Energy eqn from A to B}$$

$$\left(\frac{P_D}{\gamma} \right) + \frac{v_2^2}{2g} + 30 = \frac{P_B}{\gamma} + \frac{v_B^2}{2g} + 45 + \underbrace{\left(\frac{2v_1^2}{2g} \right)}_{(h_L)_{BD}}$$

$$\text{or, } \left(\frac{P_D}{\gamma} \right) = 15.63 \text{ m}$$

(c) EGL & HGL
A to B : Junction pts on line

$$E_A = \frac{505}{9.81} + \frac{14.08^2}{2 \times 9.81} + 75 = 136.58 \text{ m}$$

$$(h_L)_{AT} = \frac{3v_1^2}{2g} = 30.3 \text{ m}$$

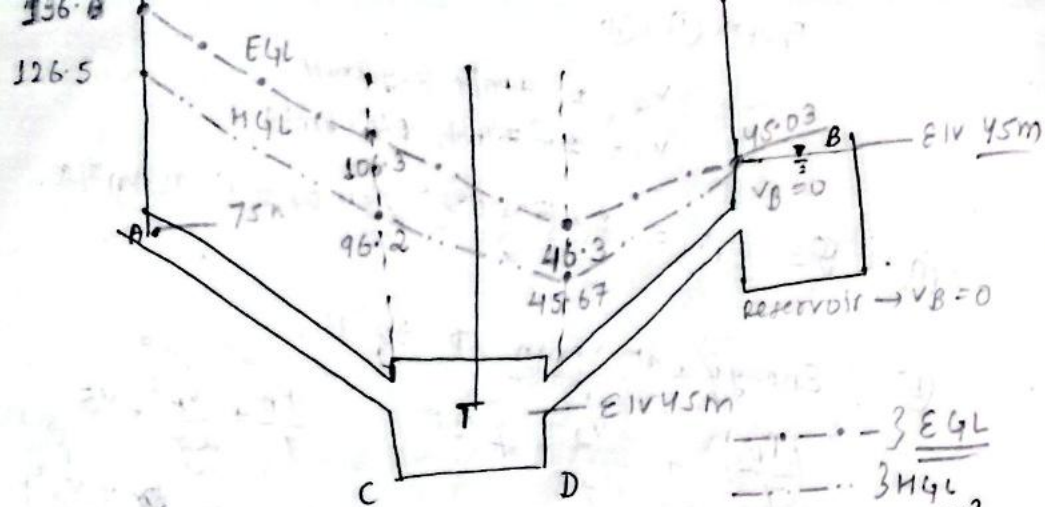
$$(h_L)_{TB} = \frac{2v_2^2}{2g} = 1.26 \text{ m}$$

$$\left\{ \begin{array}{l} \frac{v_1^2}{2g} = 10.1 \text{ m} \\ \frac{v_2^2}{2g} = 0.63 \text{ m} \end{array} \right.$$

$$h_T = 60 \text{ m}$$

$$(E_c)_{\text{just before T}} = 136.58 - 30.3 = 106.3 \text{ m}$$

$$(E_D)_{\text{just after T}} = (136.58 - 30.3) - 60 = 46.3 \text{ m}$$

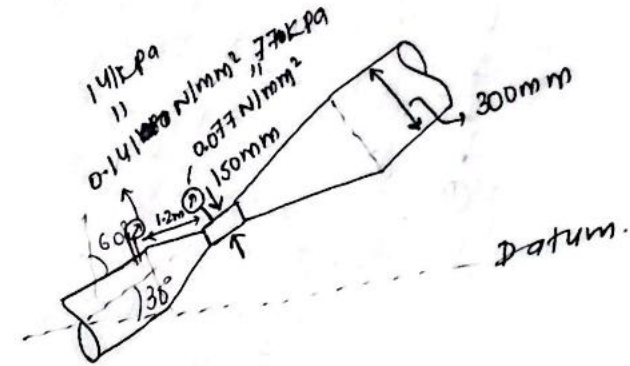


$$45.67 = \frac{P_D}{\gamma} + Z_B^{30}$$

$$\Rightarrow \frac{P_D}{\gamma} = \underline{\underline{15.67m}}$$

$$\begin{cases} HGL \rightarrow EGL - \frac{V^2}{2g} \\ E = \frac{P}{\gamma} + \frac{V^2}{2g} + Z \\ H = \frac{P}{\gamma} + Z \\ H = E - \frac{V^2}{2g} \end{cases}$$

Q 207th



$$Q = C_d \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \quad (1)$$

$$A_1 = \frac{\pi \times (300 \times 10^{-3})^2}{4} = 0.0707 \text{ m}^2$$

$$A_2 = \frac{\pi \times (150 \times 10^{-3})^2}{4} = 0.0177 \text{ m}^2$$

$$Q = \frac{2.15}{1000} = 0.00215 \text{ m}^3/\text{sec.}$$

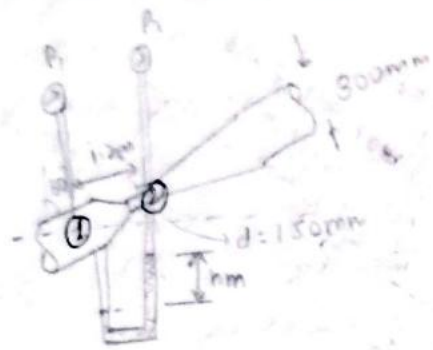
$$\text{Piezometric head } h = \left(\frac{P_1 - P_2}{\gamma} \right) + (Z_1 - Z_2) \quad \rightarrow \text{(considering datum along piezo-gauge s.t. } Z_2 = 0)$$

$$= \left(\frac{141 - 77}{0.8 \times 9.81} \right) + (0 - 1.2 \sin 30^\circ)$$

$$= \left(\frac{141 - 77}{0.82 \times 9.81} \right) + (0 - 1.2 \sin 30^\circ)$$

$$= 7.356 \text{ m}$$

$$\text{From (1), } C_d = \underline{\underline{0.979}}$$



(b) Now, If U-tube manometer is connected b/w entrance and throat, manometric reading h $\rightarrow$ 13.6 cm (say) $\rightarrow$ Non-inverted

In terms of manometer $h = \frac{h_m}{S} (1 - \frac{S_m}{S})$ $\rightarrow$ 40.82

7.358

$$\Rightarrow h_m = 0.472 \text{ m} = 472 \text{ mm}$$

If inverted, $h = \frac{h_m}{S} (1 - \frac{S_m}{S})$

(c) 7% of diff^l head is lost b/w inlet & throat. Energy eqⁿ from ① to ②

$$E_1 = E_2 + h_L$$

$$\text{or } \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \frac{7}{100} h$$

$$\text{or } \frac{141}{0.82 \times 9.81} + \frac{V_1^2}{2g} + 0 = \frac{77}{0.82 \times 9.81} + \frac{V_2^2}{2g} + 1.2 \sin 30^\circ + \frac{7}{100} \times 7.358$$

$$\text{or } V_2^2 - V_1^2 = \frac{6.84}{0.82 \times 9.81} = 134.22 \text{ --- (1)}$$

continuity eqⁿ,

$$Q = A_1 V_1 = A_2 V_2$$

$$\text{or } \frac{\pi \times 300^2}{4} \times V_1 = \frac{\pi \times 150^2}{4} \times V_2$$

$$\Rightarrow V_2 = 4V_1 \text{ --- (1)}$$

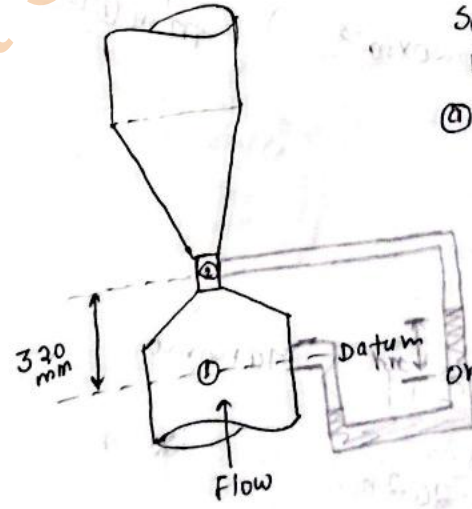
From (1) & (11)

$$V_1 = 3 \text{ m/s}$$

$$V_2 = 12 \text{ m/s}$$

$$\text{Rate of flow, } Q = A_1 V_1 = 0.212 \text{ m}^3/\text{sec}$$

(10) Solⁿ:-



$$D_2 = 100 \text{ mm}$$

$$D_1 = 200 \text{ mm}$$

$$S_o = 0.82$$

$$P_1 - P_2 = 28 \text{ KPa}$$

① Q = ?

Energy eqⁿ:

$$E_1 = E_2$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \frac{320}{1000}$$

$$\frac{P_1 - P_2}{\gamma} - \frac{320 \times 10^{-3}}{1000} = \frac{V_2^2 - V_1^2}{2g}$$

$$\frac{28}{0.82 \times 9.81} - \frac{320 \times 10^{-3}}{1000} = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow V_2^2 - V_1^2 = 57.36 \text{ --- (1)}$$

continuity equation,

$$Q = A_1 V_1 = A_2 V_2$$

$$\text{or, } \frac{\pi \times 200^2}{4} \times V_1 = \frac{\pi \times 100^2}{4} \times V_2$$

$$\Rightarrow V_2 = 4V_1 \quad \text{--- (1)}$$

From (1) & (2)

$$V_1 = 1.96 \text{ m/s}$$

$$V_2 = 7.84 \text{ m/s}$$

$$Q = A_1 V_1$$

$$= \frac{\pi \times (200 \times 10^{-3})^2}{4} \times 1.96$$

$$= 0.0616 \text{ m}^3/\text{sec.}$$

$$\textcircled{b} \quad h = \frac{P_1 - P_2}{\gamma} + (Z_1 - Z_2)$$

$$= \frac{28}{0.88 \times 9.81} + (0 - 320 \times 10^{-3})$$

$$= 2.923 \text{ m}$$

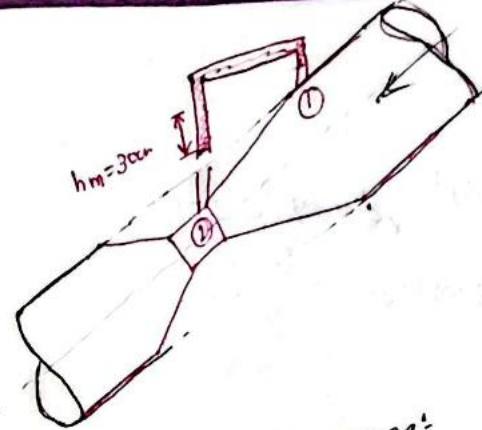
$$h_m = h \left[\frac{s_m}{s} \right]$$

$$h = h_m \left[\frac{s_m}{s} - 1 \right] \quad \text{Manome.}$$

$$\Rightarrow h_m = 0.202 \text{ m.}$$

Manometric
eqⁿ
from (1) to (2)

②



$$s_m = 0.6$$

$$h_m = 30 \text{ cm.}$$

$$(h_L)_{12} = 0.2 \times \frac{V_2^2}{2g}$$

$$V_1 \rightarrow \text{pipe.}$$

Q = ?

Energy eqⁿ:

$$E_1 = E_2 + h_L$$

$$\text{on } \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \frac{0.2 V_2^2}{2g}$$

$$\text{or, } \left[\frac{P_1 - P_2}{\gamma} \right] + (Z_1 - Z_2) = \frac{V_2^2}{2g} - \frac{0.8 V_1^2}{2g}$$

$$h = \frac{V_2^2}{2g} - \frac{0.8 V_1^2}{2g} \quad \text{--- (1)}$$

$$h = h_m \left(1 - \frac{s_m}{s} \right) \quad \text{--- (11)}$$

From (1) & (11)

$$30 \times 10^{-2} \left(1 - \frac{0.6}{0.88} \right) = \frac{V_2^2}{2g} - \frac{0.8 V_1^2}{2g}$$

$$\Rightarrow V_2^2 - 0.8 V_1^2 = 1.873 \quad \text{--- (1)}$$

continuity eqⁿ:

$$Q = A_1 V_1 = A_2 V_2 \Rightarrow V_2 = 4V_1 \quad \text{--- (11)}$$

From (1) & (11)

$$V_1 = 0.351 \text{ m/s.}$$

$$v_2 = 1.404 \text{ m/s}$$

Thus,

$$Q = A_1 v_1$$

$$= \frac{\pi \times (200 \times 10^{-3})^2}{4} \times 0.351$$

$$= 0.01103 \text{ m}^3/\text{s}$$

(11)

$$d_1 = 250 \text{ mm}$$

$$d_2 = d_0 = 150 \text{ mm}$$

$$h_m = 250 \text{ mm} \rightarrow 200 \text{ mm}$$

$$S_g = 0.9$$

$$C_d = 0.65$$

$$Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$h = h_m \left[\frac{S_m}{S} - 1 \right]$$

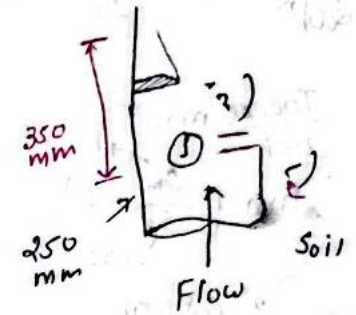
$$A_1 = 0.0491 \text{ m}^2$$

$$A_2 = 0.0177 \text{ m}^2$$

$$h = 200 \times 10^{-3} \left[\frac{18.6}{0.9} - 1 \right] = 2.822 \text{ m}$$

Thus,

$$Q = 0.0918 \text{ m}^3/\text{s}$$



Gauge deflection in manometer
 h_m
 200 mm

$$v_2 = 1.404 \text{ m/s}$$

Thus,

$$Q = A_1 v_1$$

$$= \frac{\pi \times (200 \times 10^{-3})^2}{4} \times 1.404 \times 0.35$$

$$= 0.01103 \text{ m}^3/\text{s}$$

(11)

$$d_1 = 250 \text{ mm}$$

$$d_2 = d_0 = 150 \text{ mm}$$

$$h_m = 250 \text{ mm} \quad c_d = 0.65$$

$$S_g = 0.9$$

$$Q = \frac{c_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$h = h_m \left[\frac{S_m}{S} - 1 \right]$$

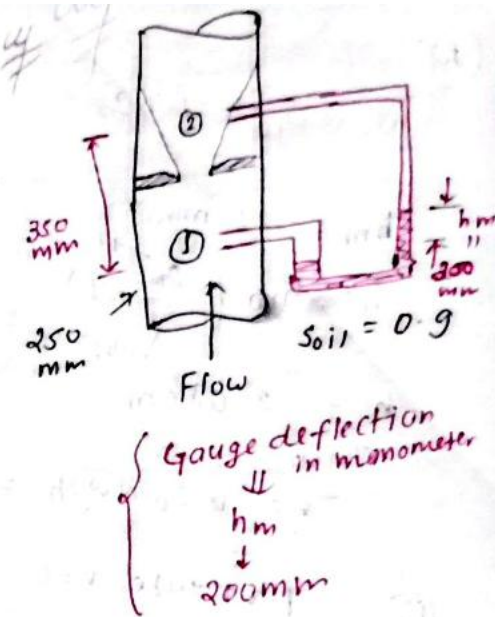
$$A_1 = 0.0491 \text{ m}^2$$

$$A_2 = 0.0177 \text{ m}^2$$

$$h = 200 \times 10^{-3} \left[\frac{18.6}{0.9} - 1 \right] = 2.822 \text{ m}$$

Thus,

$$Q = 0.0918 \text{ m}^3/\text{s}$$



(12)

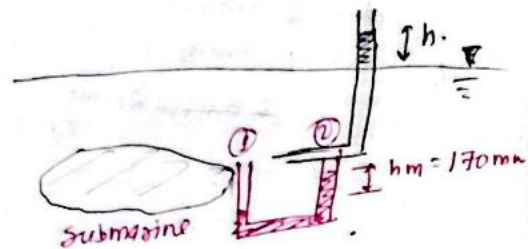
$$S_w = 1.025$$

$$h_m = 170 \text{ mm} \rightarrow 13.6$$

$$h = h_m \left(\frac{S_w}{S} - 1 \right)$$

$$h = 2.086 \text{ m}$$

$$\text{Thur, } v = C_v \sqrt{2gh} = 6.4 \text{ m/s}$$



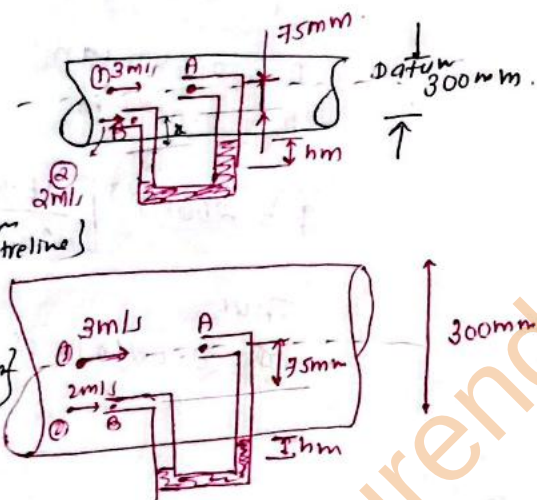
(13)

Pitot tube, $v=0$

Energy eqⁿ in ① & ②

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} = \frac{P_A}{\gamma} \left\{ \begin{array}{l} \text{Datum} \\ \text{at centreline} \end{array} \right\}$$

$$\frac{P_2}{\gamma} + \frac{v_2^2}{2g} = \frac{P_B}{\gamma} \left\{ \begin{array}{l} \text{Datum} \\ \text{at centreline} \end{array} \right\}$$



From fig,

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = 0.075 \quad \text{--- (3)}$$

From ① & ②

$$\frac{P_1 - P_2}{\gamma} = 0.075 = \frac{P_A - P_B}{\gamma \times 9810} \left[\frac{v_1^2 - v_2^2}{2g} \right]$$

$$v_1 = 3 \text{ m/s}$$

$$v_2 = 2 \text{ m/s}$$

$$\Rightarrow P_A - P_B = 3235.75 \quad \text{--- (4)}$$

Manometric eqⁿ from A to B,

$$h = h_m \left[\frac{S_m}{S} - 1 \right]$$

$$h = \frac{P_A - P_B}{\gamma} + (Z_A - Z_B)$$

$$= \frac{3235.75}{9810} + (0.075)$$

$$= 0.255 \text{ m}$$

$$h_m = \frac{0.255}{(13.6 - 1)} = 0.02022 \text{ m}$$

$$= 20.22 \text{ mm}$$

(OR)

Manometric eqⁿ from B to A with level of B as datum.

$$\frac{P_B}{\gamma} + x \gamma_w + h_m \gamma_w - \frac{h_m \gamma_m}{\gamma} - x \gamma_w - 0.075 \gamma_w = \frac{P_A}{\gamma}$$

$$h_m (\gamma_w - \gamma_m) = \frac{P_A - P_B}{\gamma} + 0.075 \times 9810$$

$$h_m = \frac{-3235.75 + 0.075 \times 9810}{9810 - 13.6 \times 9810}$$

$$= 0.02022 \text{ m}$$

$$= 20.22 \text{ mm}$$

$$P_B > P_A$$

$$P_B - P_A = 3235.75$$

(for fig with flow)

Chapter 8-9

Flow Through Orifices

Q.10.1.

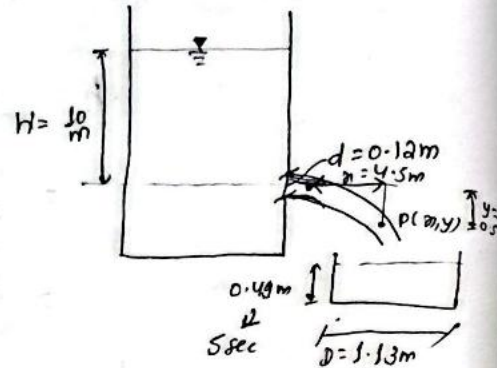
$$C_d = \frac{\text{Theoretical velocity}}{\text{Actual velocity}}$$

$$C_v = \frac{\text{Actual vel}}{\text{Theoretical vel.}}$$

$$\begin{aligned} x &= vt \\ y &= \frac{1}{2}gt^2 \\ \Rightarrow x^2 &= v^2 \left(\frac{2y}{g} \right) \\ v_{ac} &= x \sqrt{\frac{g}{2y}} \end{aligned}$$

$$v_{th} = \sqrt{2gh}$$

$$\begin{aligned} C_v &= \frac{x \sqrt{\frac{g}{2y}}}{\sqrt{2gh}} \\ &= \frac{x}{\sqrt{4hy}} \\ &= \frac{0.45}{\sqrt{4 \times 10 \times 0.54}} \\ &= 0.97 \end{aligned}$$



$$\begin{aligned} C_d &= \frac{\text{Actual discharge}}{\text{Theo. discharge}} \\ &= \frac{Q}{Q_{th}} \quad \text{orifice (or tank) } \rightarrow \text{vena-contracta} \\ Q_a &= \frac{\text{Vol}}{\text{time}} = \frac{\pi D^2 \times 0.45}{5} \\ &= 0.09828 \text{ m}^3/\text{s} \end{aligned}$$

$$\begin{aligned} Q_{th} &= A v \\ &= \left(\frac{\pi \times 0.12^2}{4} \right) \sqrt{2 \times 9.81 \times 10} \\ &= 0.158 \end{aligned}$$

$$\therefore C_d = \frac{Q_a}{Q_{th}} = 0.62$$

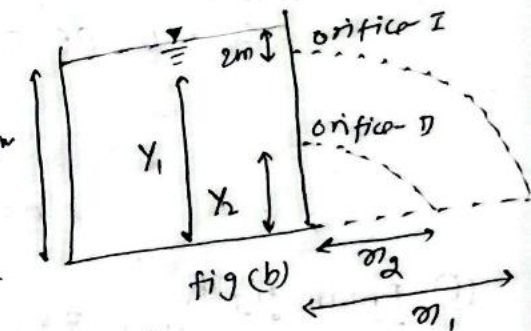
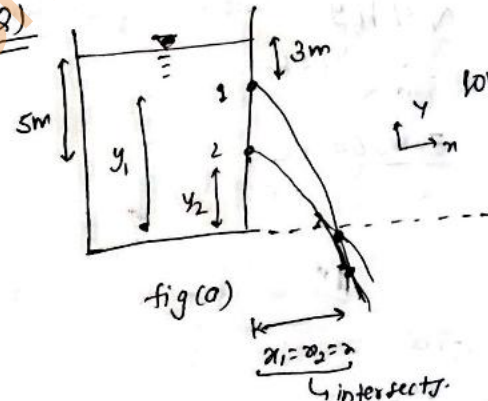
$$C_d = C_c \times C_v$$

$$\Rightarrow C_c = \frac{C_d}{C_v} = 0.641$$

$$C_c = \frac{1}{C_v^2} - 1 = 0.0672$$

$$\begin{aligned} P_{loss} &= \gamma \Phi h_{loss} \quad \text{loss of } K \cdot E \\ &= 9810 \times 0.09828 \times 10 (1 - 0.968^2) \\ &= 607.2 \text{ W} \\ &= 0.607 \text{ KW} \end{aligned}$$

(2)



$$\begin{aligned} \text{fig (a)}, \quad y_1 &= y_2 + (5-3) \\ y_1 &= y_2 + 2 \quad \text{--- (1)} \end{aligned}$$

$$\text{Identical orifices} \Rightarrow C_{v1} = C_{v2}$$

①

$$\frac{Cv_1}{\sqrt{4y_1 h_1}} = \frac{Cv_2}{\sqrt{4y_2 h_2}}$$

$x_1 = x_2 = m$ (at intersection pt.)

$$h_1 = 3m$$

$$h_2 = 5m$$

$$y_1 = \frac{5y_2}{3} \quad \text{--- (1)}$$

$$3y_1 - 5y_2 = 0 \quad \text{--- (2)}$$

from (1) & (2)

$$y_1 = 5m$$

$$y_2 = 3m$$

Thus,

$$C_v = 0.96 = \frac{x}{\sqrt{4h_1 y_1}}$$

3m 5m

$$\Rightarrow x = 7.436m$$

② From fig (b)

$$x_2 = \frac{3x_1}{4}$$

Identical orifice,

$$Cv_1 = Cv_2$$

$$\frac{y_1}{\sqrt{4y_1 h_1}} = \frac{y_2}{\sqrt{4y_2 h_2}}$$

$$\text{or, } \frac{x_1}{\sqrt{4 \times 9 \times 2}} = \frac{\frac{3x_1}{4}}{\sqrt{4 \times y_2 \times (10 - y_2)}}$$

8m

$$\text{or, } \sqrt{y_2(10 - y_2)} = 3$$

$$\text{or, } 10y_2 - y_2^2 = 9$$

$$\text{or, } y_2^2 - 10y_2 + 9 = 0$$

$$\Rightarrow y_2 = 9m$$

or

$$y_2 = 1m$$

$$\frac{A.T.P.,}{y_2 = 1m} \quad (\text{below } y_1)$$

Q. 3 (Not Pann)

(Not freely discharging)
($P < P_{atm}$:- vacuum gauge)

$$P_{atm} = 760 \times 10^{-3} \times 13.6 \times 9.81$$

$$= 101.396 \text{ kPa}$$

$$P_2 = P_{atm} = 82.407$$

$$= P_{atm} = P_{gauge}$$

$$\Rightarrow P_{gauge} = P_2 = P_{atm} - P_{vac}$$

$$= -18.99 \text{ kPa (gauge)}$$

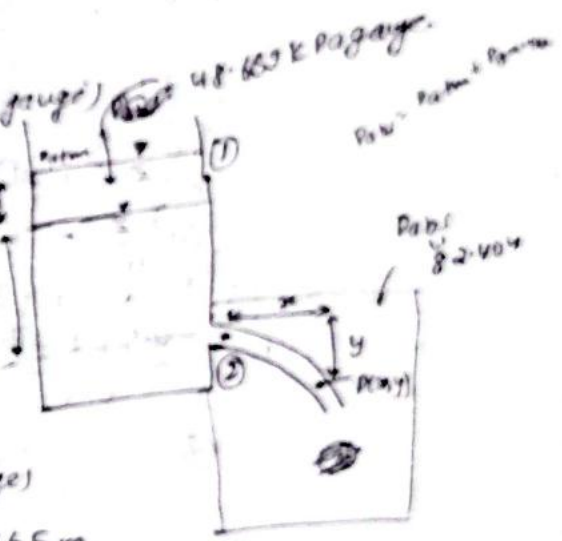
$$h' = \frac{48.069}{9.81 \times 0.85} = 5.765m$$

Energy eq: $E_1 = E_2$ (neglecting head loss)

$$\text{or, } \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + Z_2$$

(datum at orifice)

$$\text{or, } \frac{0}{\gamma} + 0 + (6.0 + 5.765) = \frac{-18.99}{9.81 \times 0.85} + \frac{v_2^2}{2g}$$



$$\frac{V_a^2}{2g} = x + 8.045 \quad \text{without considering } 10 \text{ sec}$$

Considering head loss,

$$(V_a)_{act} = V_{act} = C_v \sqrt{2gH}$$

$$\frac{(V_a)_{act}^2}{2g} = \frac{\text{Head}}{C_v^2} \quad \text{--- (1)}$$

$$\frac{C_v^2 \cdot 2g}{10 \cdot 98}$$

$$(V_a)_{act} = x \sqrt{\frac{g}{20y}}$$

$$\frac{(V_a)_{act}^2}{2g} = \frac{x^2}{4y} = \frac{1.875^2}{4 \times 0.05} = 9.453 \text{ ml}$$

So, from (1)

$$\frac{x^2}{C_v^2 \cdot 2g} = x + 8.045$$

$$\therefore \frac{9.453}{0.98^2} = x + 8.045$$

$$\Rightarrow x = 1.8 \text{ m}$$

$$(b) \text{ Vol} = \text{rate of flow thr. orifice}$$

$$Q = C_d a \sqrt{2gh}$$

$$C_d = C_v \times C_c = 0.6078$$

$$a = \frac{\pi \times (25 \times 10^{-3})^2}{4}$$

$$0.000491 \text{ m}^2$$

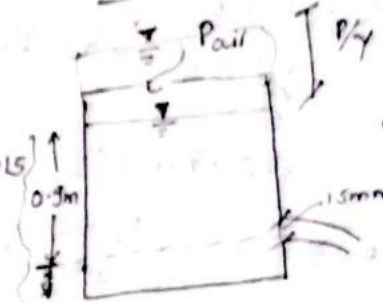
$$\text{head, } h = x + 8.045 = 9.845 \text{ m}$$

$$Q = 0.6078 \times 0.000491 \times \sqrt{2 \times 9.81 \times 9.845} = 0.00415 \text{ m}^3/\text{s}$$

(4) Solⁿ:

$$Q = 1.5 \text{ liter/sec} = 0.0015$$

$$a = \frac{\pi (15 \times 10^{-3})^2}{4} = 176.71 \times 10^{-6} \text{ m}^2$$



Pat liquid surface not Patm, so 0.9m is not actual h rather $(0.9 + \frac{p}{\gamma})$

$$Q = C_d a \sqrt{2gh}$$

$$\text{or, } 0.0015 = 0.62 \times 176.71 \times 10^{-6} \sqrt{2 \times 9.81 \times (0.9 + \frac{p}{\gamma})}$$

$$\Rightarrow p = 84894.15 \text{ N/m}^2 = 8.49 \text{ N/cm}^2$$

(5) Solⁿ:

Use
Large orifice: H_1 & H_2

$H_2 = 0.61 + 1.22 = 1.83\text{m}$
 $0.61\text{m} = H_{a1}$
 1.22m

For large orifice

$$Q_1 = \frac{2}{3} C_d \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2} \right]$$

$$= 3.241 \text{ m}^3/\text{s}$$

For small orifice,

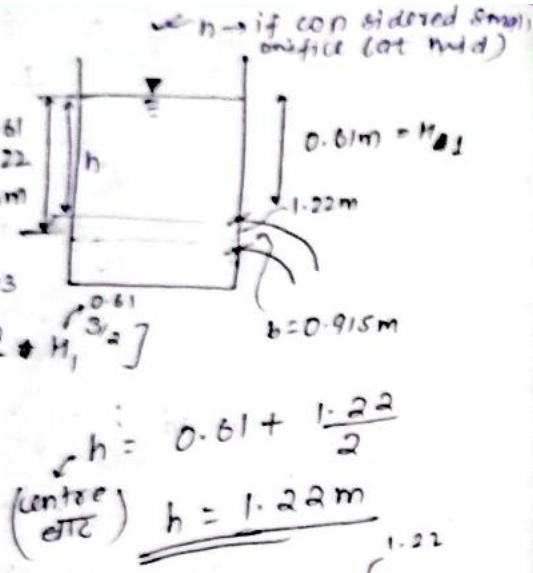
$$Q_2 = C_d A \sqrt{2gh}$$

$$= C_d \times (1.22 \times 0.915) \times \sqrt{2g \times h}$$

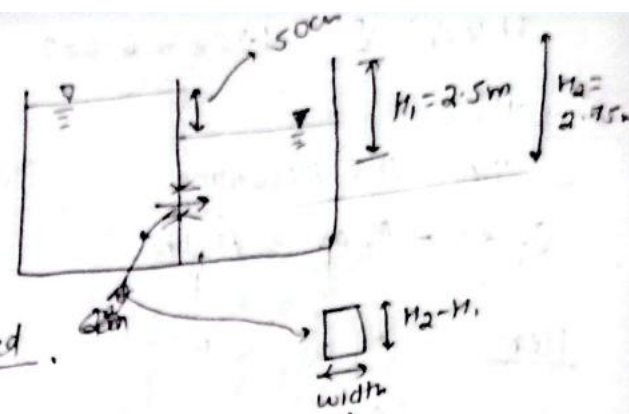
$$= 3.277 \text{ m}^3/\text{s}$$

$$\% \text{ error} = \frac{Q_2 - Q_1}{Q_1} \times 100\%$$

$$= 1.11\%$$



(6) Solⁿ:



For fully submerged

$$Q = C_d a \sqrt{2gh}$$

$$= 0.6 \times [(2.75 - 2.5) \times 2] \times \sqrt{2g \times 5.25}$$

$$= 0.9396 \text{ m}^3/\text{sec}$$

(7) Solⁿ:

$$C_d = 0.62$$

$$Q = Q_{\text{free}} + Q_{\text{submerged}}$$

$$Q_{\text{free}} = \frac{2}{3} C_d \times b \times \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2} \right]$$

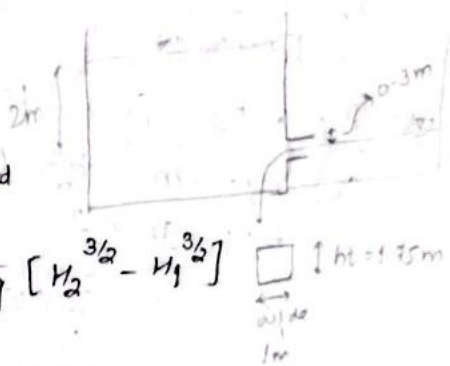
$$= \frac{2}{3} \times 0.62 \times 1 \times \sqrt{2 \times 9.81} \left[(2 + 0.3)^{3/2} - 2^{3/2} \right]$$

$$= 1.208 \text{ m}^3/\text{sec}$$

$$Q_{\text{submerged}} = C_d a \sqrt{2gh}$$

$$= 0.62 \times [(1.75 - 0.3) \times 1] \times \sqrt{2g \times 2.3}$$

$$= 6.039 \text{ m}^3/\text{sec}$$



From 1st of tank to 1st of 2nd tank

Thus, $Q = 1.208 + 6.039 = 7.247 \text{ m}^3/\text{sec}$

⑧ Solⁿ:

Just the Relation,

$$Q_T dt - Q_0 dt = A dh$$

Here, $Q_T = 0$

So,

$$Q_0 dt = -A dh$$

$$dt = -\frac{A dh}{Q_0} \quad \text{--- (1)}$$

from (1)

For cylindrical upper part,

$$t_1 = \frac{2A_1}{C_d a \sqrt{2g}} [H_1^{1/2} - H_2^{1/2}]$$

* $[H_1 \text{ \& } H_2 \text{ from level of orifice.}]$

So, $H_1 = 4\text{m}, H_2 = 1\text{m}$

$$A_1 = \frac{\pi \times 2^2}{4}$$

$$a = \frac{\pi \times (75 \times 10^{-3})^2}{4} \quad C_d = 0.62$$

$\Rightarrow t_1 = 517.88 \text{ sec}$

For lower hemispherical part,

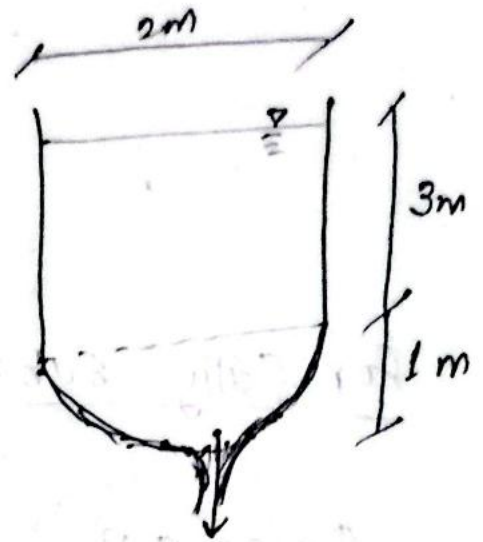
$$t_2 = \frac{2\pi}{C_d a \sqrt{2g}} \left[\frac{2}{3} R (H_1^{3/2} - H_2^{3/2}) - \frac{1}{5} (H_1^{5/2} - H_2^{5/2}) \right]$$

* $\{ H_1 = 1, H_2 = 0, R = 1\text{m} \}$

$= 241.68 \text{ sec}$

Thus,

$$t = t_1 + t_2 = 760 \text{ sec}$$



Chapter-10

Flow through Notches & Weirs

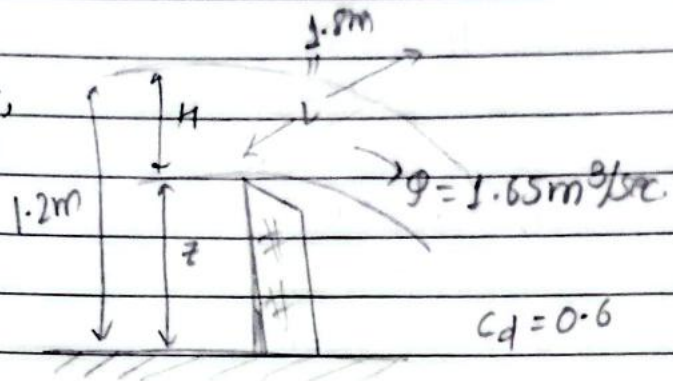
classmate

Date _____
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(1) Solution:-

For rectangular weir,
(without end corrⁿ)

$$\textcircled{1} \quad \Phi = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$



$$\text{or, } 1.65 = \frac{2}{3} \times 0.6 \times 1.8 \times \sqrt{2 \times 9.81} H^{3/2}$$

$$\Rightarrow H = 0.644 \text{ m} \quad (\text{Not considering } V_a)$$

$$Z = 1.2 - 0.644 = 0.556 \text{ m.}$$

(2) considering velocity of approach (V_a)

$$\Phi = \frac{2}{3} C_d \sqrt{2g} \left[(H + h_a)^{3/2} - h_a^{3/2} \right]$$

$$V_a = \frac{\Phi}{A_c} = \frac{1.65}{1.2 \times 1.8} = 0.764 \text{ m/s}$$

$$h_a = \frac{V_a^2}{2g} = 0.0297 \text{ m}$$

Thus,

$$1.65 = \frac{2}{3} \times 0.6 \times \sqrt{2 \times 9.81} \left[(H + 0.0297)^{3/2} - (0.0297)^{3/2} \right]$$

$$\text{or, } H = 0.827 \text{ m} \quad 0.619 \text{ m}$$

$$Z = 1.2 - 0.827 = 0.373 \text{ m.}$$

(2) Solⁿ:- $L = 0.4$

$$H = 0.225 \text{ m}$$

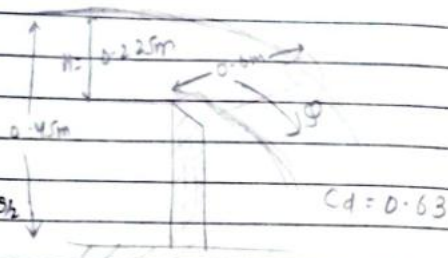
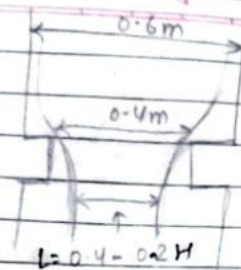
$$Z = 0.45 - 0.225$$

$$Z = 0.225 \text{ m}$$

end corrⁿ = ∞

$V_a \infty$

$Q \rightarrow$ Not given



$$Q_1 = \frac{2}{3} C_d [L - 0.1nH] \sqrt{2g} H^{3/2}$$

Q_0	$V_a = \frac{Q_0}{A_c}$ $V_a = Q_0$ 0.6×0.45	$h_a = \frac{V_a^2}{2g}$	$Q = \frac{2}{3} C_d [L - 0.1n(H+h_a)] \sqrt{2g} [(H+h_a)^{3/2} - h_a^{3/2}]$
$Q_1 = 0.0704$	0.02607	3.4×10^{-3}	$0.059 \quad 0.0719$
$Q_2 = 0.0719$	0.0266	3.61×10^{-3}	$0.157 \quad 0.0719$
$Q_3 = 0.157$			

Thus, $Q = 0.0719 \text{ m}^3/\text{sec}$

Bay $\rightarrow$ inlet of water body

(3) Solution:-

$V_a \rightarrow$ considered

$Q \rightarrow$ not given

(Table need to be made)

Head over the sill (H) = 0.75 m

Length of weir (L) = 2 m

number of end contraction

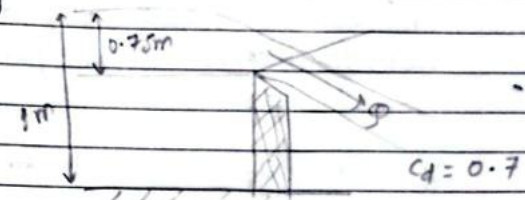
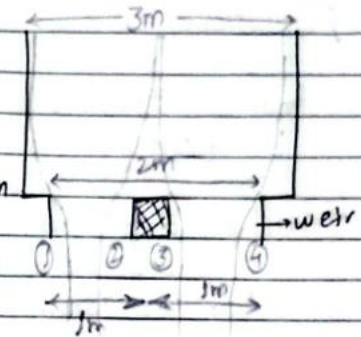
$$(n) = 4$$

$$C_d = 0.7 \text{ m}$$

length of channel (W) = 3 m

depth of channel

$$(B) = 1 \text{ m}$$



$$Q_1 = \frac{2}{3} C_d [L - 0.1nH] \sqrt{2g} H^{3/2}$$

$$= \frac{2}{3} \times 0.7 (2 - 0.1 \times 4 \times 0.75) \sqrt{2 \times 9.81} (0.75)^{3/2}$$

$$= 2.282 \text{ m}^3/\text{s}$$

Q_0	$V_a = \frac{Q_0}{A_c}$ $A_c = W \times B$ $= 3 \text{ m}^2$	$h_a = \frac{V_a^2}{2g}$	$Q = \frac{2}{3} C_d [L - 0.1n(H+h_a)] \sqrt{2g} [(H+h_a)^{3/2} - h_a^{3/2}]$
$Q_1 = 2.282$	0.7608	0.0295	2.384
$Q_2 = 2.384$	0.7946	0.0322	2.392

Q_2 & Q_3 are very close & differ by $< 1\%$

Thus,

Rate of discharge over the weir = $Q_3 = 2.392 \text{ m}^3/\text{sec}$
(considering end corrⁿ & V_a)

(4) Solution:-

$$Q_{max} = 0.7 \text{ m}^3/\text{sec}$$

1) ~~Find~~ $\theta = ?$
 $T = 1, H = ?$

For $Q = 5.6 \text{ litres/sec}$
 $= 5.6 \times 10^{-3} \text{ m}^3/\text{sec}$

$$H = 75 \text{ mm} = 0.075 \text{ m}$$

Position of apex of notch from bed of channel if max^m depth of water is not to exceed 1.5m for, $Q_{max} = 0.7 \text{ m}^3/\text{sec}$.
 $Z = ?$

C_d : same as that of a similar (in material & sharpness of edges only) right angled V-notch for which $Q = 1.407 H^{5/2}$: - (i)

$$Q_{max} = 0.7 \Rightarrow Q = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) [H]^{5/2} \quad \text{--- (ii)}$$

Equating (i) & (ii)

For, rt. angled $\theta = 90^\circ$

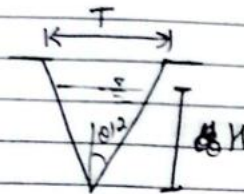
$$\frac{8}{15} \times C_d \sqrt{2g} = 1.407$$

$$\Rightarrow [C_d = 0.596]$$

$$\frac{Q = 5.6 \times 10^{-3}}{H = 0.075} = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2} \quad \text{--- (iii)}$$

$$5.6 \times 10^{-3} = 1.407 \times \tan\left(\frac{\theta}{2}\right) (0.075)^{5/2}$$

$$\Rightarrow \theta = 60.79^\circ \quad 137.68^\circ$$



For, $\theta = 60.79^\circ$, $Q_{max} = 0.7 \text{ m}^3/\text{sec}$
 From (ii),

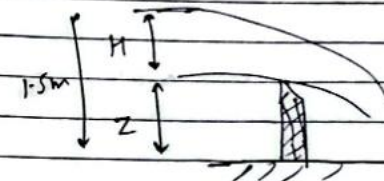
$$0.7 = 1.407 \times \tan\left(\frac{137.68^\circ}{2}\right) H^{5/2}$$

$$H = 0.517 \text{ m}$$

From triangle

$$\tan\left(\frac{\theta}{2}\right) = \frac{T}{2H}$$

$$\Rightarrow T = 2.671 \text{ m}$$



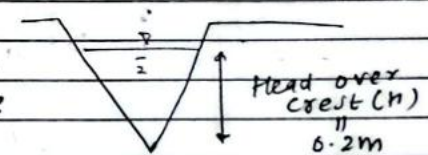
For, $H = 0.517 \text{ m}$

$$Z = 1.5 - 0.517 = 0.983 \text{ m}$$

(5) Solⁿ:-

Discharge measured within 3% accuracy.

Max V_a that can be neglected. $\rightarrow ?$



$$Q_1 = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2} \quad \left\{ \begin{array}{l} \text{Not considering } V_a \end{array} \right.$$

$$Q_2 = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) [(H + h_a)^{5/2} - h_a^{5/2}] \quad \left\{ \begin{array}{l} \text{considering } V_a \end{array} \right.$$

$$\star \left\{ \frac{Q_2 - Q_1}{Q_1 (\text{initial})} = 0.03 \right\} \text{ Condition.}$$

$$\frac{(H+h_g)^{5/2} - h_g^{5/2} - H^{5/2}}{H^{5/2}} = 0.03$$

$$\text{For } H = 0.02 \text{ m.}$$

$$(0.2 + h_g)^{5/2} - h_g^{5/2} = 0.0184$$

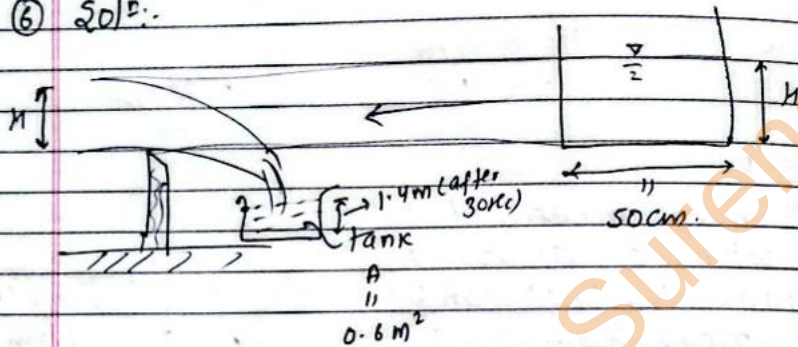
Solving,

$$h_g = 2.27 \times 10^{-3} \text{ m}$$

Thus,
corresponding V_a
 $h_g = \frac{V_a^2}{2g}$

$$\Rightarrow V_a = 0.211 \text{ m/sec.}$$

⑥ Solⁿ:



$$Q = \frac{\text{Vol}}{\text{time}} = \frac{0.6 \times 1.4}{30} = 0.028 \text{ m}^3/\text{sec.}$$

$$C_d = 0.9$$

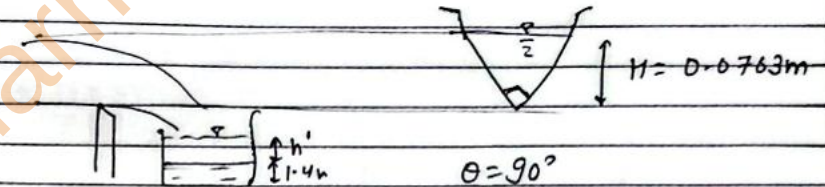
H =

Rectangular weir

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

$$\Rightarrow H = \frac{0.048 \text{ m}}{0.5} = 0.0763 \text{ m}$$

⑥ Rectangular weir replaced by 90° notch weir



$$\theta = 90^\circ$$

$$\tan\left(\frac{\theta}{2}\right) = 1$$

$$C_d = 0.8$$

$$Q = \frac{8}{15} \frac{C_d \sqrt{2g} \tan(\frac{\theta}{2})}{0.8} H^{5/2}$$

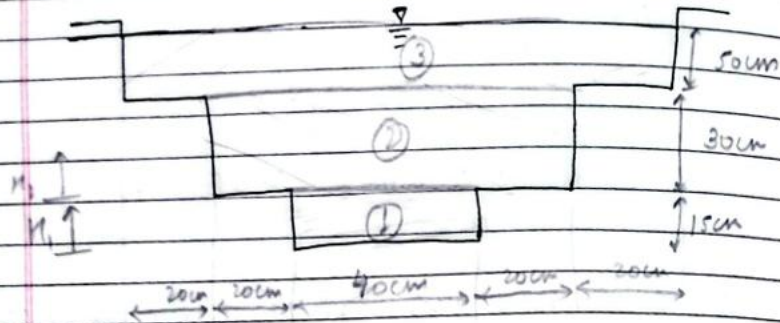
$$= 3.039 \times 10^{-3} \text{ m}^3/\text{sec.}$$

$$Q = \frac{\text{Vol}}{\text{time}} = \frac{0.6 \times h'}{30}$$

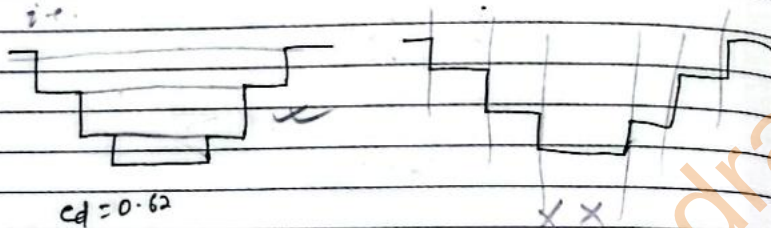
$$3.039 \times 10^{-3}$$

$$\Rightarrow h' = 0.152 \text{ m, } \left\{ \begin{array}{l} \text{After 30 sec.} \\ \text{change in depth} \\ \text{of water in} \\ \text{tank} \end{array} \right.$$

(7)



× compound Notch/Well
× section from 13 to horizontal shape
and 13 to 13 can't do vertically



$C_d = 0.62$

$$Q = \frac{2}{3} C_d L \sqrt{2g} [H_1^{3/2} - H_2^{3/2}]$$

$$\begin{aligned} Q &= Q_1 + Q_2 + Q_3 \\ &= \frac{2}{3} C_d \sqrt{2g} \left[0.4 \{ (0.95)^{3/2} - (0.80)^{3/2} \} \right. \\ &\quad \left. + 0.8 \{ (0.8)^{3/2} - (0.5)^{3/2} \} \right. \\ &\quad \left. + 1.2 \{ (0.5)^{3/2} - (0)^{3/2} \} \right] \\ &= 1.46 \text{ m}^3/\text{sec} \end{aligned}$$

(8)

Solⁿ:

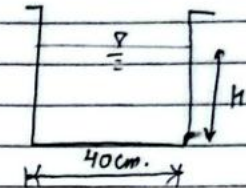
error in Head = $dH = 1.5 \text{ mm}$

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

$Q = 30 \text{ Liters/sec}$

$C_d = 0.62$

$L = 40 \text{ cm}$



$$H = 0.3705 \text{ m} = 0.118 \text{ m}$$

(a) For rectangular,

$$\frac{dQ}{Q} = 1.5 \frac{dH}{H}$$

$$= 1.5 \times 1.5 \times 10^{-3}$$

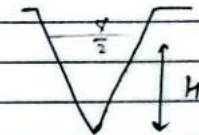
$$= 0.3705 \times 0.118$$

$$\left[\frac{dQ}{Q} = 1.89\% \right] \text{ percentage error in discharge.}$$

(b) If right-angled V-notch used,

$$Q = \frac{8}{15} C_d \tan \frac{\theta}{2} \sqrt{2g} H^{5/2}$$

$$H = 0.211 \text{ m} = 21.1 \text{ cm}$$



$dH = 1.5 \text{ mm}$

$$\frac{dQ}{Q} = 2.5 \times \frac{1.5 \times 10^{-3}}{0.211}$$

$$= 1.767\%$$

Q



a) Neglecting vel. of approach,

$$Q = 1.7 C_d L H^{3/2}$$

$\downarrow$ $\downarrow$ $\downarrow$
 0.6 40 0.4

$$= 10.32 \text{ m}^3/\text{sec}$$

b) With vel. of approach

$$Q = 1.7 C_d L (H + h_a)^{3/2}$$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
 0 0.6 40 0.4

$$h_a = \frac{v_a^2}{2g} = \frac{0.258^2}{2 \times 9.81} = 3.39 \times 10^{-3}$$

$$v_a = \frac{Q}{A_c} = \frac{10.32}{40} = 0.258 \text{ m/s}$$

$$= 10.45 \text{ m}^3/\text{sec}$$

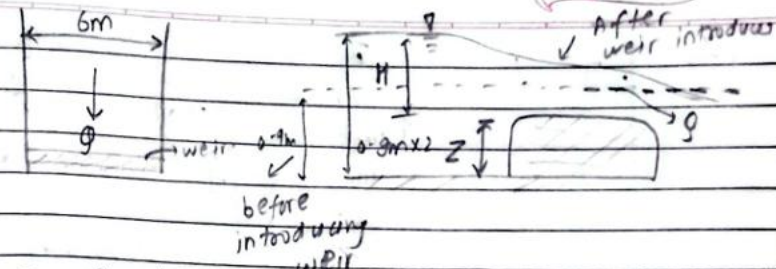
If speed power in water fall

like fall 20% - 20% / 100%

$$P = \eta \gamma Q H$$

$\downarrow$ $\downarrow$ $\downarrow$
 water (with v_a)
 &
 9810 (without v_a)

(10)



For Broad crested weir

$$Q = 1.7 C_d L H^{3/2} \quad (\text{Not considering } v_a)$$

$\downarrow$ $\downarrow$ $\downarrow$
 2.8 0.86 6m
 (same archenal)
 width
 can not mentioned
 in que.

$$H = 0.467 \text{ m}$$

$$\text{ht. of weir} = 1.9 - 0.467 = 1.333 \text{ m}$$

b) considering v_a

$$v_a = \frac{Q}{A_c} = \frac{2.8}{6 \times 1.8} = 0.26 \text{ m/s}$$

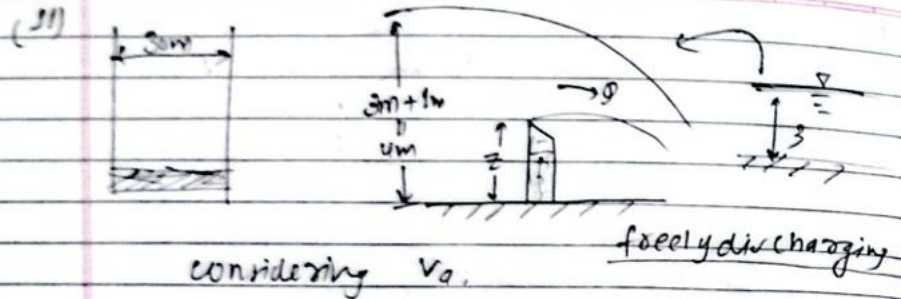
$$h_a = \frac{v_a^2}{2g} = \frac{0.26^2}{2 \times 9.81} = 3.49 \times 10^{-3}$$

$$Q = 1.7 C_d L (H + h_a)^{3/2}$$

$\downarrow$ $\downarrow$ $\downarrow$
 2.8 0.86 6

$$H = 0.4636 \text{ m}$$

$$Z = 1.8 - 0.4636 = 1.3364 \text{ m}$$



$$Q = \frac{2}{3} C_d L \sqrt{2g} [(H + h_a)^{3/2} - (h_a)^{3/2}]$$

(1)

channel: $Q = A \times V = (30 \times 3) \times 1.2 = 108 \text{ m}^3/\text{sec.}$

From (1),

For freely discharging

$$C_d = 0.58$$

$$L = 30 \text{ m}$$

$$h_a = \frac{V_a^2}{2g} = \frac{1.2^2}{2 \times 9.81} = 0.0734 \text{ m}$$

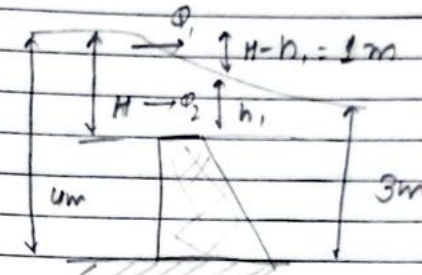
From (1) & (2)

$$108 = \frac{2}{3} \times 0.58 \times 30 \times \sqrt{2 \times 9.81} [(H + 0.0734)^{3/2} - (0.0734)^{3/2}]$$

$$H = 1.604 \text{ m}$$

$$z = 4 - 1.604 = 2.396 \text{ m}$$

Submerged



$$Q_1 = \frac{2}{3} C_d L \sqrt{2g} [(H - h_a + h_a)^{3/2} - h_a^{3/2}]$$

(3)

$$Q_2 = C_d (L \times h) \times \sqrt{2g} (H - h + h_a)$$

$$Q = Q_1 + Q_2 = 519.76 \text{ m}^3/\text{sec.}$$

$$V_a = \frac{Q}{A_c} = \frac{519.76}{4 \times 30} = 4.33 \text{ m/s}$$

Before ht. 3m $\xrightarrow{\text{water introduced}}$ ht. increased by 1m $\Rightarrow 4 \text{ m}$ (ht. of river)

$$A_c = 30 \times 3 = 90 \text{ m}^2$$

River: $Q = A \times V = 90 \times 1.2 = 108 \text{ m}^3/\text{sec.}$

freely discharging $\rightarrow 0.58$

submerged/ground discharging $\rightarrow 0.8$

Length $\rightarrow 30 \text{ m}$

(consider no end contraction)

After weir introduced,

$$Q = V_a = 9 \leftarrow 108$$

$$A_c \leftarrow 30 \times 4$$

$$= 0.9 \text{ m/s}$$

$$h_a = \frac{V_a^2}{2g} = 0.04128 \text{ m}$$

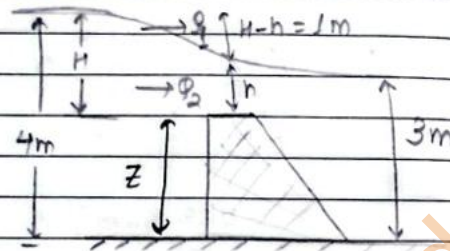
Solving (1)

$$H = 1.604 \text{ m}$$

$$Z = 2.396 \text{ m}$$

(b)

submerged $\rightarrow$ liquid in down stream.



(1) $\rightarrow$ freely discharging weir

(2) $\rightarrow$ submerged discharging orifice

$$Q = Q_1 + Q_2$$

$$108 = \frac{2}{3} C_{d1} L \sqrt{2g} \left[(H-h)^{3/2} + h_a^{3/2} \right] + \frac{2}{3} C_{d2} (L \times h) \times \sqrt{2g} (H-h+h_a)$$

$\begin{matrix} 0.58 & 30 & ? & 1\text{m} & 0.04128 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ C_{d1} & L & \sqrt{2g} & (H-h)^{3/2} & h_a^{3/2} \\ 0.58 & 30 & ? & 1\text{m} & 0.04128 \end{matrix}$

$$h = 0.496 \text{ m}$$

$$\text{Thus, } Z = 3 - 0.496 = 2.504 \text{ m}$$

(12) Cipolletti weir

$$C_d = 0.625$$

$$L = 30 \text{ m}$$

$$\text{Area of water surface of reservoir, } A = 9.3 \times 10^3 (15 + \alpha h^2)$$

$$\alpha = \text{constant} = 0.5$$

Continuity,

$$-A dh = Q dt$$

$$\text{or, } -9.3 \times 10^3 (15 + 0.5h^2) dh = \frac{9}{3} \times 0.625 \times 30 \sqrt{2g} h^{3/2} dt$$

$$t = \int_0^t dt = \int_{1.2}^{16.3} \frac{-167.97 (15 + 0.5h^2)}{h^{3/2}} dh$$

$$t = 4664.45 \text{ sec}$$

$$= 77.75 \text{ min}$$

$$= 1 \text{ hr } 17.75 \text{ min.}$$

$$\int -A dh = 1.9 L h^{3/2} dt$$

(13) Solⁿ:-

$$\text{Area of reservoir, } A = 4.65 \times 10^3 \text{ m}^2$$

For, Initial level $\rightarrow 30$

El 31.2 m to El 30.6 m

$$\text{time} = 1 \text{ hr} = 3600 \text{ sec.}$$

(1) If rectangular weir,

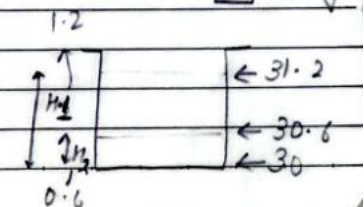
$$Q = 1.9 L H^{3/2} = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

For rectangular

$$t = \frac{3A}{C_d L \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]$$

$$3600 = \frac{1.9 \times 30 L}{2} \left[\frac{1}{\sqrt{30.6}} - \frac{1}{\sqrt{31.2}} \right]$$

$$L = 0.914 \text{ m}$$



(b) If triangular right-angled weir,
 $C_d = ?$

$$t = 3600 \text{ sec.}$$

$$A = 4.65 \times 10^3$$

$$H_1 = 1.2, H_2 = 0.6$$

$$C_d = ?$$

$$\tan(\theta/2) = 1$$

Water lower

from 9m

to 30.6m

level of crest

"
 30m.

Acc to continuity,

Vol. of liquid leaving the reservoir = Vol. of liq. flowing thr triangular weir

$$\text{or, } -A \times dh = Q dt$$

$$\text{or, } -4.65 \times 10^3 dh = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2) H^{5/2} dt$$

$H_2 \rightarrow 0.6 \quad \swarrow \quad \searrow \quad H_1$

$$\text{or, } dt = - \frac{1968.361}{C_d} \int_{H_1}^{H_2} \frac{dh}{h^{5/2}}$$

$$\text{or, } \int_0^t dt = \frac{-2 \times -1968.361}{3 C_d} \left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}} \right]$$

$\downarrow \quad \quad \quad \downarrow$
 3600 0.6 1.2

$$\text{or, } 3600 = \frac{1825.24}{C_d}$$

$$\Rightarrow C_d = 0.507$$

(1) Soln:

$$v = 1 + 2 \left(\frac{y}{y_0} \right)^{1/2}$$

$$y_0 = 3 \text{ m}$$

Energy correction factor

$$\alpha = \frac{\int_0^{y_0} v^3 dA}{V^3 A} \quad (1)$$

$$\beta = \frac{\int_0^{y_0} v^2 dA}{V^2 A} \quad (2)$$

Momentum correction factor

$$\left. \begin{aligned} dA &= b dy \\ A &= b y_0 \end{aligned} \right\} \text{we}$$

$$y_0 = 3$$

$$Q = \int_0^{y_0} v dA$$

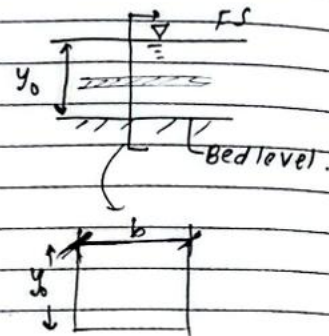
$$AV = \int_0^{y_0} v dA$$

$$V = \frac{\int_0^{y_0} v dy}{y_0} = \frac{1}{3} \int_0^3 \left[1 + 2 \left(\frac{y}{3} \right)^{1/2} \right] dy$$

$$= \frac{1}{3} \left[y + \frac{2}{\sqrt{3}} \times \frac{2}{3} y^{3/2} \right]_0^3$$

$$= \frac{7}{3} \text{ m/s}$$

Avg. mean velocity



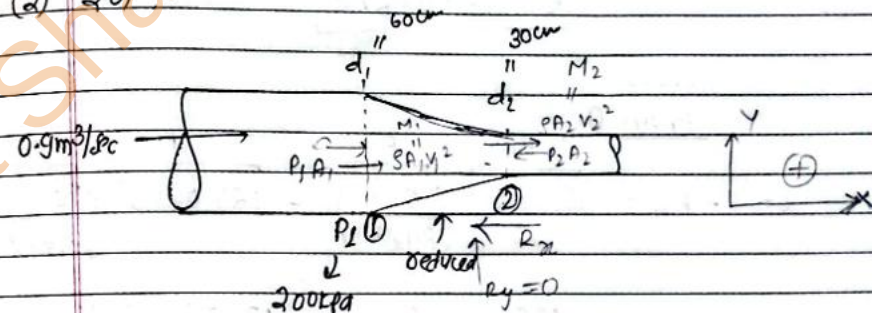
$$\alpha = \frac{\int_0^3 \left[1 + 2 \left(\frac{y}{3} \right)^{1/2} \right]^3 b dy}{\left(\frac{7}{3} \right)^3 \times b \times 3}$$

$$= 1.118$$

$$\beta = \frac{\int_0^3 \left[1 + 2 \left(\frac{y}{3} \right)^{1/2} \right]^2 b dy}{\left(\frac{7}{3} \right)^2 \times b \times 3}$$

$$= 1.04$$

(2) Soln:



wt not given $\rightarrow$ neglect

$$\therefore, P_y = 0$$

$$\sum F_x = dM = M_2 - M_1$$

$$\text{or, } -R_x + P_1 A_1 - P_2 A_2 = \rho A_2 v_2^2 - \rho A_1 v_1^2$$

$$= \rho Q (v_2 - v_1) \quad (1)$$

$$v_2 = \frac{Q}{A_2} = \frac{0.9}{\pi \times 0.3^2} = 12.73 \text{ m/s}$$

$$v_1 = \frac{Q}{A_1} = \frac{0.9}{\pi \times 0.6^2} = 3.183 \text{ m/s}$$

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$Q = 0.9 \text{ m}^3/\text{sec}$
For P_2 ,
energy equation,

$$E_1 = E_2 + h_L$$

$$\text{or } \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\text{or } \frac{200}{9.81} + \frac{3.183^2}{2 \times 9.81} = \frac{P_2}{9.81} + \frac{12.73^2}{2 \times 9.81}$$

$$P_2 = 124.04 \text{ KPa}$$

Now,

From ①

$$\text{or } -R_x + \frac{P_1 A_1}{\gamma} - \frac{P_2 A_2}{\gamma} = 0$$

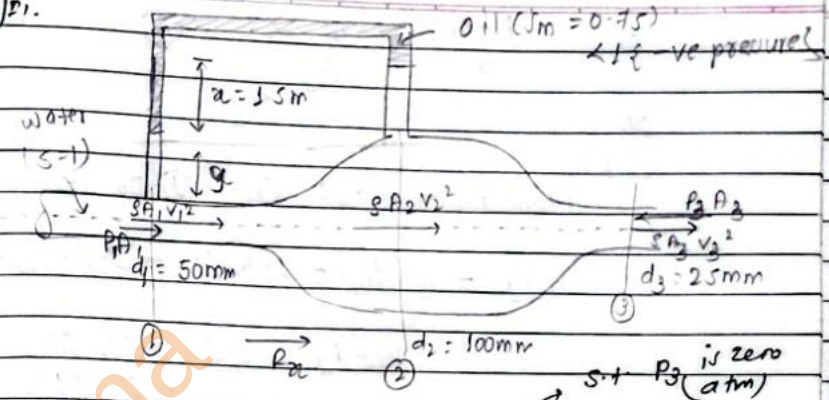
$$= \frac{1000 \times 0.9}{9} \times (12.73 - 3.183)$$

$$\Rightarrow R_x = 39188.6 \text{ N} (\leftarrow)$$

$$R_y = 0$$

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(3) Soln.



Momentum equation, in ③ & ①

$$\Sigma F_x = dM = M_2 - M_1 = M_3 - M_1$$

Step ①

$$\text{or } R_x + P_1 A_1 - P_3 A_3 = \rho A_3 V_3^2 - \rho A_1 V_1^2$$

$P_3 = 0$, then P_1 comes as Gauge pressure
w.r.t. atm
(P_3)_{atm} = 0

$$\checkmark P_1 = ?$$

$$\checkmark A_1 = \frac{\pi \times (50 \times 10^{-3})^2}{4} = 1.96 \times 10^{-3} \text{ m}^2$$

$$\checkmark P_3 = 0$$

$$\checkmark A_3 = \frac{\pi \times (25 \times 10^{-3})^2}{4} = 4.908 \times 10^{-4} \text{ m}^2$$

$$\checkmark V_3 = V_1$$

U-tube manometer.

Step 2: Energy eqⁿ in ① & ②

$Z_1 = Z_2 = Z_3 \rightarrow$ horizontal pipe

if ① & ③ can't find. (more unknowns)

$$E_1 = E_2 + h_L$$

$$\text{or } \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\text{or, } \frac{P_1 - P_2}{\gamma} = \frac{v_2^2 - v_1^2}{2g} \quad \text{--- (2)}$$

or

Step 3:

Manometric eqⁿ from (1) to (2),

$$P_1 - \gamma x \gamma_w - 1.5 \times \gamma_0 + 1.5 \times \gamma_w + \gamma \gamma_w = P_2$$

$$\text{or, } P_1 - P_2 = 1.5 \times 0.75 \times 9810 - 1.5 \times 1 \times 9810$$

$$\frac{P_1 - P_2}{\gamma} = 1.5 \left(\frac{\gamma_0}{\gamma} - \frac{\gamma_w}{\gamma} \right)$$

$$= 1.5 \left(\frac{800}{8} - 1 \right)$$

$$= 1.5 \left(\frac{0.75}{1} - 1 \right)$$

$$= -0.375$$

So, eqⁿ (2)

$$v_1^2 - v_2^2 = 7.3575 \quad \text{--- (3)}$$

Step 4: Continuity eqⁿ.

$$A_1 v_1 = A_2 v_2$$

$$\text{or, } \frac{\pi \times 50^2}{4} \times v_1 = \frac{\pi \times 100^2}{4} \times v_2$$

$$\text{or, } v_1 = 4 v_2 \quad \text{--- (4)}$$

From (3) & (4),

$$v_2 = 0.7 \text{ m/s}$$

$$v_1 = 2.8 \text{ m/s}$$

$$\text{Also, } A_1 v_1 = A_3 v_3$$

$$\text{or, } \frac{\pi \times 50^2}{4} \times 2.8 = \frac{\pi \times 25^2}{4} \times v_3$$

$$\Rightarrow v_3 = 11.2 \text{ m/s}$$

Energy eqⁿ in (1) & (3)

$$\frac{P_1 - P_3}{\gamma} = \frac{v_3^2 - v_1^2}{2g}$$

$$\text{or, } P_1 = \frac{11.2^2 - 2.8^2}{2 \times 9.81} \times 9810$$

$$P_1 = 58.8 \text{ kN/m}^2$$

Now,

From (1)

$$R_m = \rho A_3 v_3^2 - \rho A_1 v_1^2 - P_1 A_1 + P_3 A_3$$

$$= 1000 \times 4.908 \times 10^{-4} \times 11.2^2$$

$$- 1000 \times 1.96 \times 10^{-3} \times 2.8^2 -$$

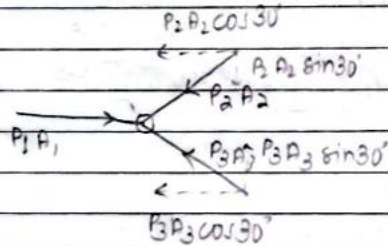
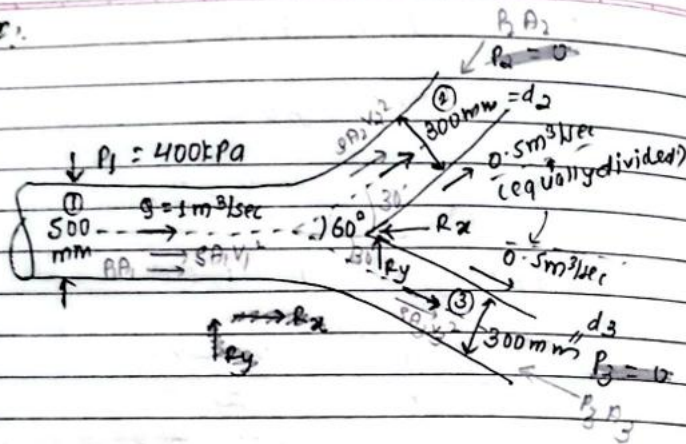
$$58.8 \times 10^3 \times 1.96 \times 10^{-3}$$

$$= -69.05 \text{ N}$$

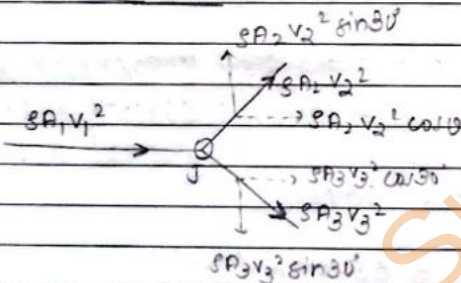
$$\therefore [R_m = 69.05 \text{ N (} \leftarrow \text{)}]$$

↑
Total pull on bolts.

(4) Solⁿ:



Pressure Force



Momentum flux

* Wt NOT given $\Rightarrow$ Neglect

Junction कहेसी

Junction बिपर

$$\sum F_x = dM_x = \frac{(M_2 + M_3) - (M_1)_x}{\Delta t}$$

$$\text{Or, } -R_x - P_2 A_2 \cos 30^\circ - P_3 A_3 \cos 30^\circ + P_1 A_1 = (\rho A_2 V_2^2 \cos 30^\circ + \rho A_3 V_3^2 \cos 30^\circ) - \rho A_1 V_1^2$$

L(1)

$$Q = 1 \text{ m}^3/\text{sec} = A_1 V_1 = \pi \times 0.5^2 \times V_1$$

$$\Rightarrow V_1 = 10.18 \text{ m/s}$$

$$Q_3 = Q_2 = 0.5 \text{ m}^3/\text{sec} = A_2 V_2 = A_3 V_3 = \frac{\pi \times 0.3^2}{4} \times V_2$$

$$\Rightarrow V_2 = V_3 = 14.15 \text{ m/s}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$A_1 = \frac{\pi \times 0.5^2}{4} = 0.196 \text{ m}^2$$

$$A_2 = A_3 = \frac{\pi \times 0.3^2}{4} = 0.0707 \text{ m}^2$$

$$\sum F_y = dM_y = (M_2 + M_3)_y - (M_1)_y$$

$$\text{Or, } R_y - P_2 A_2 \sin 30^\circ + P_3 A_3 \sin 30^\circ = (\rho A_2 V_2^2 \sin 30^\circ - \rho A_3 V_3^2 \sin 30^\circ) \quad \text{--- (2)}$$

* In both (1) & (2) P_2 & P_3 unknown, use energy equation

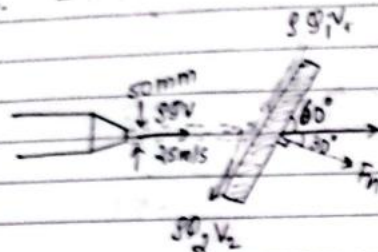
In (1) & (2),

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$P_1 = 400 \text{ kPa}, \gamma = 9810, V_1 = 10.18 \text{ m/s}, V_2 = 14.15 \text{ m/s}$$

$$[P_2 = 351.7 \text{ kPa}]$$

Q Soln:-



Discharge in
tangential dir, $Q_1 = ?$, $Q_2 = ?$

Horizontal thrust, $F_x = ?$

Vertical thrust, $F_y = ?$

Normal thrust, $F_n = ?$

Dirⁿ of F_R , $\alpha = ?$

Work done, $W = ?$

eff. of jet, $\eta = ?$

(a) When plate is stationary,

$$Q_1 + Q_2 = Q, \quad Q_1 - Q_2 = Q \cos 60^\circ$$

$$Q = AV$$

$$= \frac{\pi \times 0.05^2 \times 25}{4}$$

$$= 0.0491 \text{ m}^3/\text{sec.}$$

$$Q_1 + Q_2 = 0.0491$$

$$Q_1 - Q_2 = 0.0245$$

Solving.

$$A = \frac{\pi \times 0.05^2}{4} \times 1.96 \times 10^3$$

(b) When

$$Q_1 = 0.0368 \text{ m}^3/\text{sec}$$

$$Q_2 = 0.0123 \text{ m}^3/\text{sec.}$$

$$F_n = \rho Q V \sin^2 \theta$$

$$= \rho AV V \sin^2 \theta$$

$$= 1000 \times 1.96 \times 10^{-3} \times 25^2 \times \sin^2(60^\circ)$$

$$= 918.75 \text{ N}$$

$$F_y = \rho Q V \sin \theta \cos \theta$$

$$= 1000 \times 1.96 \times 10^{-3} \times 25^2 \times \sin(60^\circ) \cos(60^\circ)$$

$$= 530.44 \text{ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= 1060.88 \text{ N}$$

$$\alpha = \tan^{-1} \left(\frac{F_y}{F_x} \right) = 30^\circ$$

Work done, $W = 0 \text{ N-m}$ { Stationary plate }

$$\text{Efficiency, } \eta = \frac{W}{E} = 0\%$$

(c) When plate is moving with velocity of 10 m/s in direction of jet

$$u = 10 \text{ m/s}$$

$$V_R = V - u = 25 - 10 = 15 \text{ m/s}$$

Thus.

$$Q_1 + Q_2 = Q = AV_R = 1.96 \times 10^{-3} \times 15 =$$

$$Q_1 + Q_2 = 0.0294$$

$$Q_1 - Q_2 = 0.0147$$

Solving

$$Q_1 = 0.022 \text{ m}^3/\text{sec}$$

$$Q_2 = 0.00735 \text{ m}^3/\text{sec.}$$

$$F_n = \rho Q V_R \sin^2 \theta$$

$$= \rho A V_R^2 \sin^2 \theta$$

$$= 1000 \times 1.96 \times 10^{-3} \times 15^2 \times \sin^2(60^\circ)$$

$$= 330.75 \text{ N}$$

$$F_y = \rho Q V_R \sin \theta \cos \theta$$

$$= \rho A V_R^2 \sin \theta \cos \theta$$

$$= 190.96 \text{ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= 381.92 \text{ N}$$

$$\alpha = \tan^{-1} \left(\frac{190.96}{330.75} \right)$$

$$= 30^\circ$$

Work done, $W = F_R \times u$

$$= 381.92 \times 330.75 \times 10$$

$$= 12621.6 \text{ N-m}$$

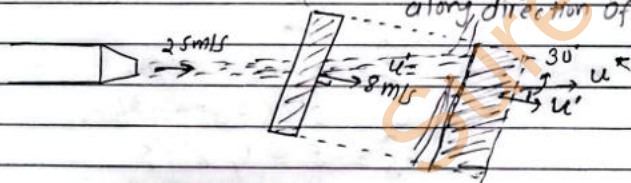
$$= 12621.6 \text{ N-m/s}$$

$$E = \frac{1}{2} m V^2 = \frac{1}{2} \rho A V^3$$

$$= 15312.5 \text{ N-m}$$

$$\eta = \frac{W}{E} = \frac{12621.6}{58247.3} = 21.6\%$$

© When plate is moving with velocity of 8 m/s parallel to itself and in direction normal to surface. (All problems same but use the velocity of plate along direction of jet)



$$Q = A V_r$$

$$V_r = 25 - 9.24 = 15.76 \text{ m/s}$$

Along direction of jet,

$$u = u' = 8 \cos 30^\circ$$

$$u = 9.24 \text{ m/s}$$

$$Q_1 + Q_2 = 1.96 \times 10^{-3} \times 15.76 = 0.0309$$

$$Q_1 - Q_2 = 0.015$$

Solving,

$$Q_1 = 0.0229 \text{ m}^3/\text{sec}$$

$$Q_2 = 0.0079 \text{ m}^3/\text{sec}$$

$$F_x = \rho Q V_r \sin^2 \theta$$

$$= 1000 \times 0.0309 \times 15.76 \sin^2(60^\circ)$$

$$= 364.056 \text{ N}$$

$$F_y = \rho Q V_r \sin \theta \cos \theta$$

$$= 210.188 \text{ N}$$

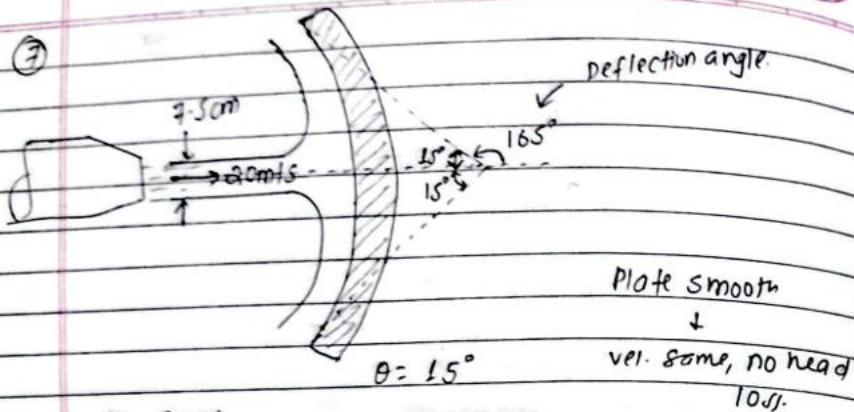
$$F_R = \sqrt{F_x^2 + F_y^2} = 420.38 \text{ N}$$

$$\alpha = \tan^{-1} \left(\frac{F_y}{F_x} \right) = 30^\circ$$

$$W = F_R \times u$$

$$= 420.38 \times 9.24 = 3883.87 \text{ N-m/s}$$

$$\eta = \frac{W}{E} = \frac{3883.87}{\frac{1}{2} \rho A V^3} = 21.97\%$$



② Rest

$$F_y = 0$$

$$F_x = F_n = F_p = \rho Q V (1 + \cos \theta)$$

$$= \frac{1000 \times \pi \times 0.075^2}{4} \times 20^2 (1 + \cos 15^\circ)$$

$$= 13474.07 \text{ N}$$

$$\text{Work done per second on plate, } W = F_x \times u$$

$$= 0 \text{ Nm/s}$$

$$\text{Power of jet, } = W = 0 \text{ kW}$$

$$\eta = \frac{W}{E} = 0\%$$

⑥ when curve plate is moving with 8 m/s in direction of jet.

$$F_y = 0 \text{ N}$$

$$F_x = F_n = F_p = \rho Q V_r (1 + \cos \theta)$$

$$= \frac{1000 \times \pi \times 0.075^2}{4} \times (20 - 8)^2$$

$$\times (1 + \cos 15^\circ)$$

$$= 1250.67 \text{ N}$$

$$W_1 = F_n \times u$$

$$= 1250.67 \times 8$$

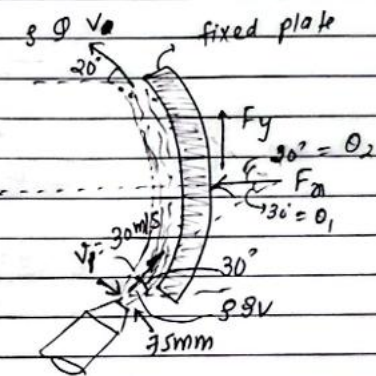
$$= 10005.34 \text{ Nm/s}$$

$$P = W = 10 \text{ kW}$$

$$\eta = \frac{W}{E} = \frac{10005.34}{\frac{1}{2} \times 1000 \times \frac{\pi \times 0.075^2}{4} \times 20^3}$$

$$= 56.62\%$$

⑧



$$F_x = \rho Q V (\cos \theta_1 + \cos \theta_2)$$

$$= \frac{1000 \times \pi \times 0.075^2}{4} \times 30^2 (\cos 30^\circ + \cos 20^\circ)$$

$$= 7179.68 \text{ N}$$

$$F_y = \rho Q V (\sin \theta_2 - \sin \theta_1)$$

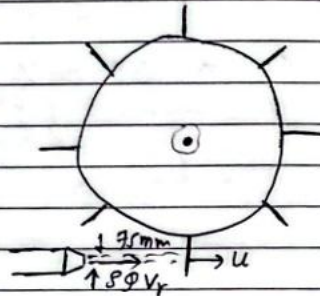
$$= \frac{1000 \times \pi \times 0.075^2}{4} \times 30^2 [\sin 20^\circ - \sin 30^\circ]$$

$$= -628.14 \text{ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2} = 7207.1 \text{ N}$$

$$\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 5^\circ$$

⑨ Soln:



* $\Phi = AV \rightarrow$ For turbine, For any
 $v \rightarrow$ vel. of jet. C.A. of turbine

$$F_y = 0 \text{ N}$$

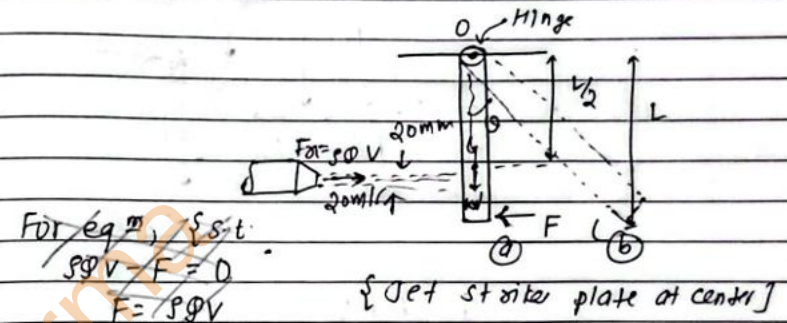
$$\begin{aligned} F_h = F_R = F_x &= \rho \Phi V_r \\ &= 1000 \times \pi \times 0.075^2 \times 20 \times (20 - 5) \\ &= 1325.36 \text{ N} \end{aligned}$$

$$W = F_x \times u = 1325.36 \times 5 = 6626.8 \text{ N-m/s}$$

$$\begin{aligned} \eta &= \frac{W}{E} = \frac{6626.8}{\left\{\frac{1}{2} \rho AV v^3\right\}} \rightarrow \text{jet exit velocity} \\ &= 37.5\% \quad \text{energy} \end{aligned}$$

(10) $\rightarrow$ Soln:

uniform thickness $\rightarrow$ C.G. lies at center



$$\begin{aligned} \text{For eq. } \sum F &= 0 \\ \rho \Phi V - F &= 0 \\ F &= \rho \Phi V \end{aligned}$$

{ Jet strikes plate at center }

$$\text{a) } \sum M_O = 0 \quad \{ F \text{ s.t. plate's verticle} \}$$

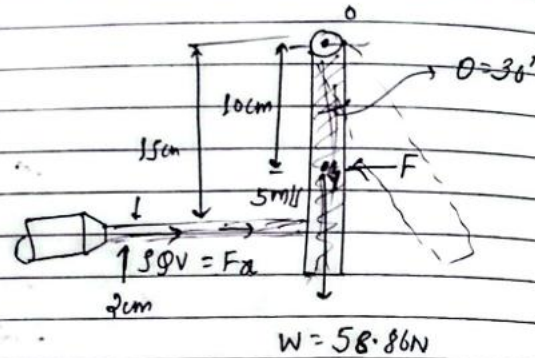
$$F \times L - F_x \times \frac{L}{2} = 0$$

$$\begin{aligned} F &= \frac{F_x}{2} = \frac{\rho \Phi V}{2} \\ &= 1000 \times \frac{\pi \times 0.02^2}{4} \times 20^2 \\ &= 62.83 \text{ N} \end{aligned}$$

b) When plate is allowed to swing freely, $\theta = ?$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{\rho \Phi V}{W}\right) \\ &= \sin^{-1}\left(\frac{62.83 \times 5}{150}\right) \\ &= 56.9^\circ \end{aligned}$$

(31) Solution:



$$Q = A \times V$$

$$= \frac{\pi \times 0.02^2 \times 5}{4}$$

$$= 8.14 \times 10^{-4} \times 5$$

$$= 1.57 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\Sigma M_o = 0$$

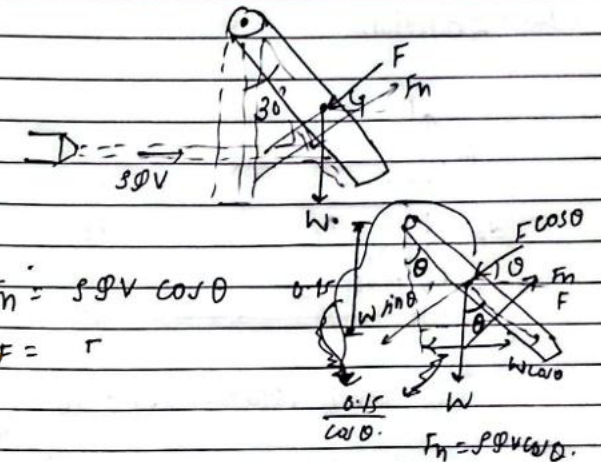
$$F_n \times 15 - F \times 10 = 0$$

$$F = \frac{3 \rho Q V}{2}$$

$$= \frac{3 \times 1000 \times 1.57 \times 10^{-3} \times 5}{2}$$

$$F = 11.78\text{N}$$

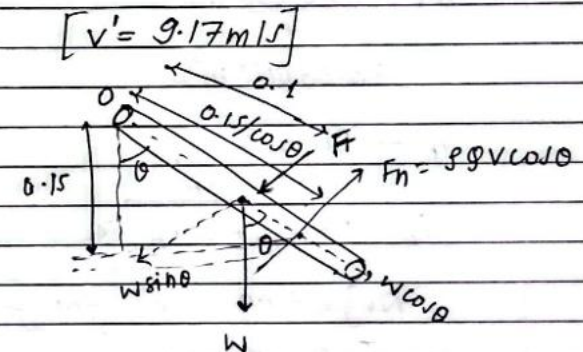
When, plate deflected thr. 30° , s.t.
same force continues to act at the
centre of gravity of plate.
 $\theta = 30^\circ$



$$\Sigma M_o = 0$$

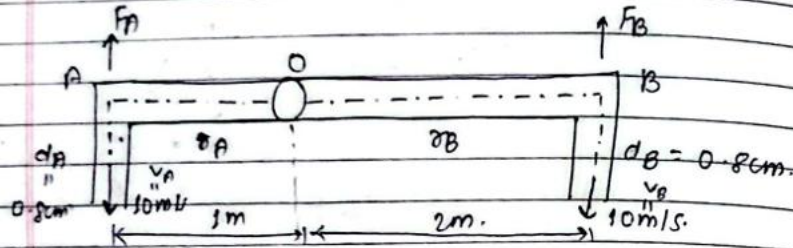
$$F \cos \theta \times 0.1 + W \sin \theta \times 0.1 - \rho Q V \cos \theta \times 0.15 = 0$$

$$11.78 \times \cos 30^\circ + 58.86 \times \sin 30^\circ - \rho Q V \cos 30^\circ \times 0.15 = 0$$



(32) → Solution:-

✓ जो दि. है ✓ को opp. dir. में Force act जे



(a) arm stationary: $\omega = 0$

$$d_A = d_B = 0.8 \text{ cm}$$

$$v_{Ay} = v_{By} = 10 \text{ m/s}$$

$$r_A = 1 \text{ m}$$

$$r_B = 2 \text{ m}$$

$$Q = AV = \frac{\pi \times 0.008^2 \times 10}{4}$$

$$= 5.03 \times 10^{-4} \text{ m}^3/\text{s}$$

$v_A' \rightarrow$ Absolute

$$F_A = \rho Q (v_{Ay} + \omega r_A) r_A$$

Neglecting ωr , for finding dir.:-

$$T_A = \rho Q v_{Ay} r_A$$

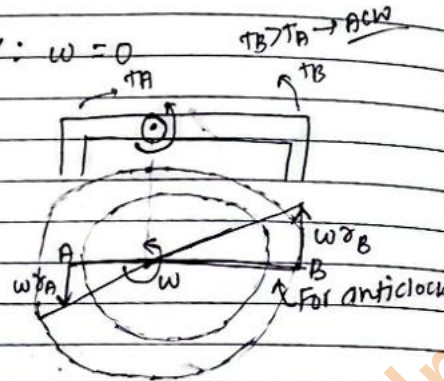
$$= 1000 \times 5.03 \times 10^{-4} \times 10 \times 1$$

$$= 5.03 \text{ Nm}$$

$$T_B = \rho Q v_{By} r_B$$

$$= 1000 \times 5.03 \times 10^{-4} \times 10 \times 2$$

$$= 10.06 \text{ Nm}$$



As, $T_B > T_A$ - So it rotate anticlockwise.

AC/W

Absolute velocity,

$$v_A' = v_A + \omega r_A \quad \{ v_A \text{ \& } \omega r_A \text{ both in same dir.} \}$$

$$v_A' = 10 + \omega$$

$$v_B' = v_B - \omega r_B = 10 - 2\omega$$

Now

$$\text{Total Torque} = T_B - T_A$$

$$= \rho Q v_B' r_B - \rho Q v_A' r_A$$

$$= \rho Q [v_B' r_B - v_A' r_A]$$

$$= \rho Q [(v_B + \omega) r_B]$$

$$= \rho Q \left[\underbrace{(10 - 2\omega) r_B}_{T_2} - \underbrace{(10 + \omega) r_A}_{T_1} \right]$$

(a) For stationary rotating arm,

$$\omega = 0$$

$$T = \rho Q [10 \times 2 - 10 \times 1]$$

$$= 5.03 \text{ Nm}$$

(b) For arm to free to rotate

$$T = 0$$

$$\text{or, } 0 = \rho Q [(10 - 2\omega) 2 - (10 + \omega)]$$

$$\text{or, } 20 - 4\omega - 10 - \omega = 0$$

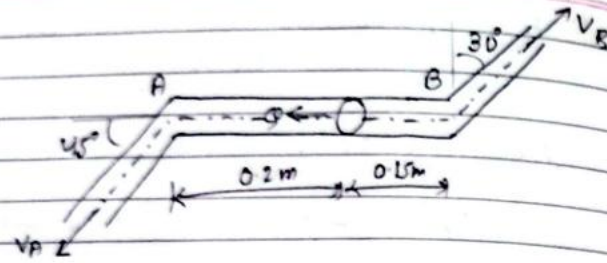
$$\text{or, } \omega = \frac{10}{5} = 2 \text{ rad/sec}$$

Speed of rotation, $N = ?$

$$\omega = \frac{2\pi N}{60}$$

$$N = \frac{2 \times 60}{2\pi} = 19.098 \text{ rpm}$$

(13) Soln.



$$d_A = d_B = 5 \text{ mm}$$

$$r_A = 0.2 \text{ m}$$

$$r_B = 0.15 \text{ m}$$

By top. $\rightarrow Q = 6 \text{ lt/min} = \frac{6 \times 10^{-3}}{60} = 10^{-4} \text{ m}^3/\text{s}$

$$v_A = v_B = \frac{Q_A/Q_B}{A_A/A_B} = \frac{10^{-4}}{\pi \times 0.005^2}$$

$\rightarrow$ By both sprinkler $Q_A = Q_B$ { since same dia }

$$Q_A = Q_B = \frac{Q}{2} = 5 \times 10^{-5} \text{ m}^3/\text{s}$$

So,

$$v_A = v_B = \frac{5 \times 10^{-5}}{\pi \times 0.005^2} = 2.55 \text{ m/s}$$

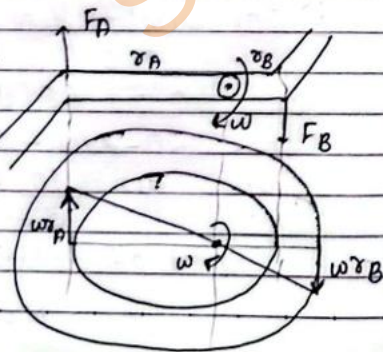
$$v_{Ay} = v_A \sin 45^\circ = 1.80 \text{ m/s}$$

$$v_{By} = v_B \cos 30^\circ = 2.21 \text{ m/s}$$

$\hookrightarrow F_A$ & F_B opp. to vel.

$\times$ Both combinely rotates clockwise.

So, no need to calculate T_A & T_B .



Total torque, $T = T_A + T_B$

$$= \rho Q_A v_A' r_A + \rho Q_B v_B' r_B$$

$$T = \rho Q_A [(v_{Ay} - \omega r_A) r_A + (v_{By} - \omega r_B) r_B]$$

For stationary sprinkler,

$$\omega = 0$$

$$T = \rho Q_A [v_{Ay} r_A + v_{By} r_B] \quad (\rho_A = \rho_B)$$

$$= 1000 \times 5 \times 10^{-5} [1.8 \times 0.2 + 2.21 \times 0.15]$$

$$= 0.37 \text{ Nm} = 0.0346 \text{ Nm (C/W)}$$

For free rotation, $T = 0$, $\omega = ?$

$$\text{or. } 0 = \rho Q_A [(1.8 - 0.2\omega) 0.2 + (2.21 - 0.15\omega) 0.15]$$

$$\text{or. } 0.36 - 0.04\omega + 0.3315 - 0.0225\omega = 0$$

$$\text{or. } \omega = \frac{0.6915}{0.04 + 0.0225}$$

$$\omega = 11.064 \text{ rad/sec}$$

(1) Solⁿ:-

velocity profile

$$\frac{u}{U} = \sin\left(\frac{\pi}{2} \frac{y}{\delta}\right)$$

Laminar boundary, flat plate, zero pressure gradient

$$\frac{\partial P}{\partial x} = 0$$

Energy correction factor, $\alpha = ?$

Momentum correction factor, $\beta = ?$

$\delta < BL$

$\delta < BL$ Thickness

$$\alpha = \frac{\int u^3 dA}{V \bar{u}^3 A}$$

$$\beta = \frac{\int u^2 dA}{V \bar{u}^2 A}$$

$V \bar{u} \rightarrow$ Mean velocity (Continuity/mass rate)

$$\dot{Q} = A \bar{u} V = \int u dA$$

$$\text{Within BL} \int V \bar{u} = \frac{\int u dA}{A} = \int_0^\delta u \sin\left(\frac{\pi}{2} \frac{y}{\delta}\right) dx dy$$

Flat plate: say, thickness 1

'or'

$$A = \text{width} \times \delta$$

$$dA = dy \times 1$$

For BL,

For small strip

$$dA = dy \times \text{width}$$

$$V = \frac{1}{\delta} U \left[-\cos\left(\frac{\pi}{2} \frac{y}{\delta}\right) \right]_0^\delta$$

$$V = \frac{0 - \pi}{2\pi} \frac{2U}{\pi} \text{ (Average velocity)}$$

$$\alpha = \frac{\int_0^\delta U^3 \sin^3\left(\frac{\pi}{2} \frac{y}{\delta}\right) dy \times 1}{\frac{8 U^3}{\pi^3} \times \delta \times 1}$$

$$= \frac{8 \pi^3}{8 \pi^3 \delta} \int_0^\delta \sin^3\left(\frac{\pi}{2} \frac{y}{\delta}\right) dy$$

$$\int \sin^3(x) dx = -\cos x + \frac{\cos^3 x}{3} + C$$

$$= \frac{2\delta}{\pi} \int_0^\delta \sin^3\left(\frac{\pi y}{2\delta}\right) dy \times \frac{\pi}{2\delta}$$

$$= \frac{2\delta}{\pi} \left[-\cos\left(\frac{\pi y}{2\delta}\right) + \frac{\cos^3\left(\frac{\pi y}{2\delta}\right)}{3} \right]_0^\delta$$

$$= \frac{2\delta}{\pi} \left[-0 + \frac{1}{3} \times 0 + 1 - \frac{1}{3} \right]$$

$$= 0.4244\delta$$

Thus,

$$\Delta = \frac{\pi^3 \times 0.4244\delta}{8\pi^2}$$

$$= 1.65 \text{ m}$$

$$\sin^3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\beta = \int_0^\delta U^2 \sin^2\left(\frac{\pi y}{2\delta}\right) dy$$

$$\left(\frac{2U}{\pi}\right)^2 \times \delta \times 1$$

$$= \frac{\pi^2}{8\pi^2} \int_0^\delta \left[\frac{1 - \cos\left(\frac{2\pi y}{\delta}\right)}{2} \right] dy$$

$$= \frac{\pi^2}{8\pi^2} \left[y - \frac{\sin\left(\frac{\pi y}{\delta}\right)}{\pi/\delta} \right]_0^\delta$$

$$= \frac{\pi^2}{8\pi^2} [\delta]$$

$$= 1.23 \text{ m}$$

(2) Q1:-

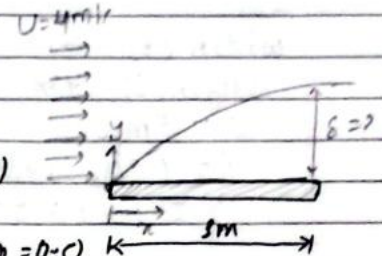
$$L = 1 \text{ m}$$

$$U = 4 \text{ m/s}$$

Flat plate

(a) δ at end of plate = ? ($x = 1$)

(b) τ at middle of plate = ? ($x = 0.5$)



$$\text{Kinematic viscosity, } \nu = 0.15 \times 10^{-4} \text{ m}^2/\text{s}$$

$$\rho_{\text{air}} = 1.226 \text{ kg/m}^3$$

For flat plate,

$$(Re)_a = ?$$

$$(Re)_a = \frac{Ux}{\nu} = ?$$

$$\text{For, } x = 1 \text{ m, } (Re)_a = \frac{4 \times 1}{0.15 \times 10^{-4}}$$

$$= 2.67 \times 10^5$$

$$(< 3 \times 10^5)$$

i.e. it is laminar.

Thus,

$$(a) \text{ At, end } \delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 1}{\sqrt{2.67 \times 10^5}} = 9.67 \text{ mm}$$

(b) Shear stress,

$$\tau_0 = \frac{1}{2} \rho C_d U^2$$

$$Re_x < 3 \times 10^5 \text{ laminar}$$

$$\text{So, } C_d = \frac{0.664}{\sqrt{Re_x}} = 1.82 \times 10^{-3}$$

If asked shear force,
 $F = \tau \times A_c$

$$(Re)_{0.5} = \frac{4 \times 0.5}{0.15 \times 10^{-4}} = 1.33 \times 10^5$$

$$\tau_0 = \frac{1}{2} \times 1.226 \times 1.82 \times 10^{-3} \times 4^2$$

$$= 0.0178 \text{ N/m}^2$$

③ Solⁿ:

width (b) = 2m

length (L) = 4m

$U = 5 \text{ m/s}$

$\rho = 1.208 \text{ kg/m}^3$

$\nu = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$

For laminar,

$Re_x = 3 \times 10^5$

$Re_x = \frac{Ux}{\nu}$

$[x = L = 0.882 \text{ m}]$

Boundary layer
ceases to be
laminar

just
After
 Re_x
3x10⁵

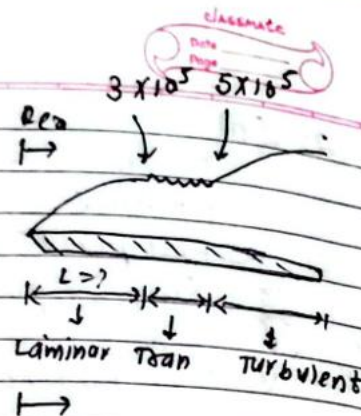
$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.882}{\sqrt{3 \times 10^5}} = 8.05 \times 10^{-3} \text{ m} = 8.05 \text{ mm}$$

If not said
(bed shear
stress)

$$\tau = \frac{1}{2} \rho C_d U^2$$

$$C_d = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{\sqrt{3 \times 10^5}} = 1.21 \times 10^{-3}$$

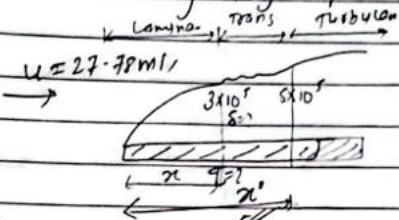
$$\tau = \frac{1}{2} \times 1.21 \times 10^{-3} \times 1.208 \times 5^2 = 0.0183 \text{ N/m}^2$$



(4) upto which boundary layer persists.

①

$U = 100 \text{ km/hr}$
 $= 27.78 \text{ m/s}$



$$Re_x = \frac{Ux}{\nu}$$

$\nu = 1.49 \times 10^{-5} \text{ m}^2/\text{s}$

$$3 \times 10^5 = \frac{27.78 x}{1.49 \times 10^{-5}}$$

$$[x = 0.161 \text{ m}]$$

Laminar sublayer
doesn't exist in
laminar zone
↓
it exists in
turbulent layer/zone

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.161}{\sqrt{3 \times 10^5}} = 1.47 \text{ mm}$$

For, $\rho = 1.226 \text{ kg/m}^3$

$$\tau = \frac{1}{2} \times 1.226 \times \left(\frac{0.664}{\sqrt{3 \times 10^5}} \right) \times 27.78^2$$

$$= 0.573 \text{ N/m}^2$$

✓ Laminar sublayer doesn't exist

② $Re_x = 5 \times 10^5$ (Turbulent) → where turbulent
BL starts.

$$\delta = \frac{0.376x'}{(Re_x)^{1/5}}$$

$$Re_x = \frac{Ux'}{\nu}$$

$$\Rightarrow [x' = 0.268 \text{ m}]$$

$$[\delta = 7.3 \text{ mm}]$$

$$\tau_0 = \frac{1}{2} C_D \rho U^2$$

$$C_D = \frac{0.059}{(Re_x)^{1/5}}$$

$$[\tau_0 = 2.023 \text{ N/m}^2]$$

For laminar sub-layer thickness,

$$\delta' = \frac{11.6 \nu}{u_x}$$

$$u_x = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{2.023}{1.226}} = 1.285$$

$$\delta' = \frac{11.6 \times 1.49 \times 10^{-5}}{1.285} = 0.135 \text{ mm}$$

(5) Solution

Given velocity distribution,
 $u = ay^2 + by + c$

$a, b, c = ?$ (Boundary condition)

displacement thickness, δ^*

Momentum " "

Boundary condition

(i) for $y=0$, $u=0$ so, $c=0$

(ii) for $y=\delta$, $u=U$ so, $U = a\delta^2 + b\delta - 0$

(iii) At $y=\delta$, $\frac{du}{dy} = 0$

$$\left. \frac{du}{dy} \right|_{y=\delta} = 2a\delta + b = 0 \quad \text{--- (iv)}$$

Solving (i) & (iv)

$$\begin{array}{ccccccc} \delta^2 & \delta & & & & & \\ 2\delta & 1 & & & & & \\ & & a & & b & & \end{array}$$

$$[a=U], \quad b = -2\delta U$$

$$a\delta^2 + b\delta = 0$$

$$\begin{array}{ccc} a & b & \\ \delta^2 & \delta & \\ 2\delta & 1 & \end{array}$$

velocity distribution law

$$u = Uy^2 - 2\delta Uy$$

Displacement thickness, $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

$$= \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= \left[y - \frac{y^2}{\delta} + \frac{y^3}{3\delta^2} \right]_0^\delta$$

$$= \left(\delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} \right)$$

$$= \frac{\delta}{3}$$

Momentum thickness, θ

$$\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right) \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2} \right) dy$$

$$= \int_0^\delta \left(\frac{2\delta y - y^2}{\delta^2} \right) \left(\frac{\delta^2 - 2\delta y + y^2}{\delta^2} \right) dy$$

$$= \frac{1}{\delta^4} \int_0^\delta [2\delta^3 y - 4\delta^2 y^2 + 2\delta y^3 - y^2 \delta^2 + 2\delta y^3 - y^4] dy$$

$$= \frac{1}{\delta^4} \left[\frac{2\delta^3 y^2}{2} - \frac{4\delta^2 y^3}{3} + \frac{2\delta y^4}{4} - \frac{\delta^2 y^3}{3} + \frac{2\delta y^4}{4} - \frac{y^5}{5} \right]_0^\delta$$

$$= \frac{2}{15} \delta$$

⑥ Solution:

velocity dist. in laminar boundary layer

$$\frac{u}{U} = a + bm + cm^2 + dm^3$$

$U \rightarrow$ free stream vel.

$$m = \left(\frac{y}{\delta}\right)$$

$$\frac{u}{U} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y^2}{\delta^2}\right) + d\left(\frac{y^3}{\delta^3}\right)$$

Boundary condition

① At, $y=0, u=0, a=0$

② At, $y=\delta, u=U$
 $b + c + d = 1 - \text{①}$

③ At, $y=\delta, \frac{du}{dy} = 0$

$$\frac{du}{dy}\bigg|_{y=\delta} = \frac{b}{\delta} + \frac{2cy}{\delta^2} + \frac{3dy^2}{\delta^3} = 0$$

$$\Rightarrow b + 2c + 3d = 0 - \text{④}$$

④ At, $y=0, \frac{d^2u}{dy^2} = 0$

$$\frac{d^2u}{dy^2}\bigg|_{y=0} = \frac{2c}{\delta^2} + \frac{6dy}{\delta^3} = 0$$

$$2c + 6d = 0 - \text{⑤}$$

Solving ①, ② & ③

$$a=0, b = \frac{3}{2}, c=0, d = -\frac{1}{2}$$

$$\text{So, } \frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y^3}{\delta^3}\right)$$

displacement thickness, $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

$$= \int_0^\delta \left[1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y^3}{\delta^3}\right)\right] dy$$

$$= \left[y - \frac{3y^2}{4\delta} + \frac{1}{8} \frac{y^4}{\delta^3}\right]_0^\delta$$

$$= \frac{3\delta}{8} \text{ or}$$

⑦ Soln:

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$$

$$\delta^* = \int_0^\delta \left[1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] dy$$

$$= \left[y - \frac{3}{2} \frac{y^2}{\delta} + \frac{1}{6} \frac{y^3}{\delta^2}\right]_0^\delta$$

$$= \frac{5}{12} \delta$$

Thus, $\left[\frac{\delta^*}{\delta} = \frac{5}{12}\right]$

$$\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \left[1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] dy$$

$$= \int_0^\delta \left[\frac{3\delta y - y^2}{2\delta^2}\right] \left[\frac{\delta^2 - 3\delta y + y^2}{2\delta^2}\right] dy$$

$$= \frac{1}{4\delta^4} \int_0^\delta [3\delta^3 y - 9\delta^2 y^2 + 3\delta y^3 - y^2 \delta^2 + 3\delta y^3 - y^4] dy$$

Shape factor $\rightarrow \frac{\delta^*}{\delta}$ & $\frac{\theta}{\delta}$ classmate
Date _____
Page _____

$$= \frac{19}{120} \delta$$

$$\left[\frac{\theta}{\delta} = \frac{19}{120} \right]$$

⑧ Solⁿ:

$$\frac{u}{U} = \frac{3}{2} \left(\frac{y}{\delta} \right)^2 - \frac{1}{2} \left(\frac{y}{\delta} \right)^3$$

$U \rightarrow$ vel. at distance y from surface.
 $y \rightarrow \delta, u \rightarrow U.$

$$\begin{aligned} \delta^* &= \int_0^\delta \left(1 - \frac{3}{2} \frac{y^2}{\delta^2} + \frac{1}{2} \frac{y^3}{\delta^3} \right) dy \\ &= \left[y - \frac{y^3}{2\delta^2} + \frac{y^4}{8\delta^3} \right]_0^\delta \\ &= \frac{5\delta}{8} \end{aligned}$$

$$\begin{aligned} \theta &= \int_0^\delta \left(\frac{3}{2} \frac{y^2}{\delta^2} - \frac{1}{2} \frac{y^3}{\delta^3} \right) \left(1 - \frac{3}{2} \frac{y^2}{\delta^2} + \frac{1}{2} \frac{y^3}{\delta^3} \right) dy \\ &= \int_0^\delta \frac{(3\delta y^2 - y^3)(\delta^3 - 3\delta y^2 + y^3)}{2\delta^3 \times 2\delta^3} dy \\ &= \frac{1}{4\delta^6} \int_0^\delta [3\delta^4 y^2 - 9\delta^2 y^4 + 3\delta y^5 - y^3 \delta^3 + 3\delta y^5 - y^7] dy \\ &= \frac{1}{4\delta^6} \left[\frac{3\delta^4 y^3}{3} - \frac{9\delta^2 y^5}{5} + \frac{3\delta y^6}{6} - \frac{y^4 \delta^3}{4} + \frac{3\delta y^6}{6} - \frac{y^7}{7} \right]_0^\delta \\ &= \frac{39}{280} \delta \end{aligned}$$

Shape factor $\left(\frac{\delta^*}{\delta}, \frac{\theta}{\delta} \right)$ classmate
Date _____
Page _____

80 Shear stress at surface,

$$\begin{aligned} \tau &= \mu \frac{du}{dy} \\ &= \mu \left(\frac{3y}{\delta^2} - \frac{3}{2} \frac{y^2}{\delta^3} \right) U \end{aligned}$$

Boundary condition,

$$\begin{aligned} \text{for } y &= \delta \\ &= \mu \left(\frac{3\delta}{\delta^2} - \frac{3}{2} \frac{\delta^2}{\delta^3} \right) U \\ &= \frac{3}{2} \mu U \end{aligned}$$

⑨ Solⁿ:

$$\frac{u}{U} = \left(\frac{y}{\delta} \right)^{1/7}$$

$$\begin{aligned} \delta^* &= \int_0^\delta \left[1 - \left(\frac{y}{\delta} \right)^{1/7} \right] dy \\ &= \left[y - \frac{(y)^{1/7+1}}{(\frac{1}{7}+1)\delta^{1/7}} \right]_0^\delta \\ &= \frac{1}{8} \delta \end{aligned}$$

$$\Rightarrow \frac{\delta^*}{\delta} = 0.125$$

$$\begin{aligned} \theta &= \int_0^\delta \left(\frac{y}{\delta} \right)^{1/7} \left(1 - \left(\frac{y}{\delta} \right)^{1/7} \right) dy \\ &= \frac{1}{\delta^{2/7}} \int_0^\delta [\delta^{1/7} y^{1/7} - y^{2/7}] dy \\ &= \frac{1}{\delta^{2/7}} \left[\frac{\delta^{1/7} y^{(1/7+1)}}{(\frac{1}{7}+1)} - \frac{y^{(2/7+1)}}{(\frac{2}{7}+1)} \right]_0^\delta \\ &= 0.097 \delta \end{aligned}$$

$\left[\frac{\theta}{\delta} = 0.097\right]$

Energy thickness

$$\delta_E = \int_0^{\delta} \frac{u}{U} \left(1 - \left(\frac{u}{U}\right)^2\right) dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/7} \left(1 - \left(\frac{y}{\delta}\right)^{2/7}\right) dy$$

$$= \int_0^{\delta} \left[\left(\frac{y}{\delta}\right)^{1/7} - \left(\frac{y}{\delta}\right)^{3/7}\right] dy$$

$$= \left[\frac{y^{(8/7)}}{\frac{8}{7} \delta^{1/7}} - \frac{y^{(10/7)}}{\frac{10}{7} \delta^{3/7}} \right]_0^{\delta}$$

$$= \frac{7}{40} \delta$$

$$\left[\frac{\delta_E}{\delta} = 0.175\right]$$

If at certain section, $U = 10 \text{ m/s}$, $\delta = 25 \text{ mm}$,
 $\rho = 1.226 \text{ kg/m}^3$

Energy loss per unit length,

$$E_L = \frac{1}{2} \rho U^3 \delta_E \quad [\delta_E = 0.175 \delta]$$

$$= \frac{1}{2} \times 1.226 \times 10^3 \times 10^3 \times 0.175 \times 25 \times 10^{-3}$$

$$= 2.682 \text{ Nm/s}$$

$$\frac{1}{2} \rho U^3 \delta_E$$

$$= \frac{1}{2} \rho U^3 \left(\frac{\delta_E}{\delta}\right) \delta$$

$$= \frac{1}{2} \rho U^3 \delta_E$$

(10) Soln:

$$U = 11 \text{ m/s}$$

$$\delta = ? \quad \delta^* = ? \quad \theta = ?$$

$$\rho = 1.2 \text{ kg/m}^3$$

$$\mu = 17 \times 10^{-6} \text{ N s/m}^2$$

In laminar region.

$$\delta = \frac{5x}{\sqrt{Re_x}} \quad \text{for laminar region.}$$

But can't put $Re_x = 3 \times 10^5$ at position in laminar region unknown

$$Re_x = \frac{Ux}{\nu}$$

$$\delta = \frac{5x}{\sqrt{\frac{Ux}{\nu}}} = \frac{5\sqrt{\nu x}}{\sqrt{U}} = 0.00567 \sqrt{\nu x}$$

$$\delta = 0.00567 \sqrt{\nu x} = 56.74 \times 10^{-4} \sqrt{\nu x} \text{ m}$$

For Laminar, Max x ,

$$Re_x = 3 \times 10^5 = \frac{11 \times x \times 1.2}{17 \times 10^{-6}}$$

$$\Rightarrow [x = 0.386 \text{ m}] \rightarrow \text{for laminar}$$

Displacement thickness, for laminar, $\delta^* = \frac{\delta}{3}$

Momentum thickness for laminar, $\theta = \frac{\delta}{8}$

$x(\text{m})$	0	0.1	0.2	0.3	0.386
$\delta(\times 10^{-4})$	0	17.94	25.37	31.07	35.25
$\delta^*(\times 10^{-4})$	0	5.98	8.46	10.36	11.75
$\theta(\times 10^{-4})$	0	2.24	3.17	3.88	4.4

II) Solⁿ.

First determine $\rightarrow$ laminar or turbulent

$$D = 2R = 100 \text{ mm}$$

$$R = 50 \text{ mm}$$

$$S = 0.8$$

$$\nu = 0.0004 \text{ m}^2/\text{s}$$

$$\text{a) } Q_1 = 0.65 \text{ lps} = 650 \times 10^{-6} \text{ m}^3/\text{s}$$

$$\text{b) } Q_2 = 130 \text{ lps} = 130 \times 10^{-3} \text{ m}^3/\text{s}$$

$$Q_1 = A V_1 \Rightarrow V_1 = \frac{Q_1}{A} = \frac{650 \times 10^{-6}}{\pi \times (50 \times 10^{-3})^2} = 0.0827 \text{ m/s}$$

$$Q_2 = A V_2 \Rightarrow V_2 = \frac{Q_2}{A} = 16.55 \text{ m/s}$$

For pipe flow, Laminar $\Rightarrow (Re)_{\max} = 2000$
turbulent $\Rightarrow (Re) \rightarrow > 4000$

$$\text{Reynolds number, } Re = \frac{V D}{\nu}$$

$$Re_1 = \frac{V_1 D}{\nu} = \frac{0.0827 \times 100 \times 10^{-3}}{0.0004} = 20.675$$

$$Re_2 = \frac{V_2 D}{\nu} = \frac{16.55 \times 100 \times 10^{-3}}{0.0004} = 4137.5$$

For, $Re_1 = 20.675$ Laminar
entrance length, $L_e = 0.058 Re_1 D$
 $= 0.12 \text{ m}$

For, $Re_2 = 4137.5$ turbulent

entrance length, $L_e = (50-80) Re_2 D$
Say, 65,
 $L_e = 26893.75 \text{ m}$

Chapter-13

Submerged body

I) solution.

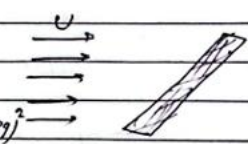
$$U = 50 \text{ km/hr} = 13.89 \text{ m/s}$$

$$A = 2 \text{ m} \times 1 \text{ m} = 2 \text{ m}^2$$

$$\gamma_{\text{air}} = 11.282 \text{ N/m}^3 \quad \gamma = 99$$

$$\Rightarrow \rho = 1.15 \text{ kg/m}^3$$

$$C_L = 0.75 \quad C_D = 0.15 \quad \text{for given angle}$$

$$\text{a) } F_L = \frac{1}{2} C_L \rho A U^2$$


$$= \frac{1}{2} \times 0.75 \times 1.15 \times 2 \times (13.89)^2$$

$$= 166.4 \text{ N}$$

$$\text{b) } F_D = \frac{1}{2} C_D \rho A U^2$$

$$= 33.28 \text{ N}$$

$$\text{c) Resultant force } (F) = \sqrt{F_L^2 + F_D^2}$$

$$= 169.7 \text{ N}$$

$$\text{d) Direction } \theta = \tan^{-1} \left(\frac{F_L}{F_D} \right) = 78.69^\circ$$

e) Power exerted by air stream on plate,

$$P = F_D \times U = 462.26 \text{ Watt}$$

② Solⁿ:-

$$A = 6.5 \text{ m}^2$$

$$U = 70 \text{ km/hr} = 19.44 \text{ m/s}$$

$$\text{Total force, } F_T = 2000 \text{ N}$$

$$F_{\text{rolling}} = 20\% \text{ of } F_T = 400 \text{ N}$$

$$F_{\text{surface friction}} = 10\% \text{ of } F_T = 200 \text{ N}$$

$$F_{\text{form/pressure drag}} = \text{Remaining} = 1400 \text{ N}$$

$$C_{DP} = ? \rightarrow \text{Due to form drag.}$$

$$\rho_{\text{air}} = 1.25 \text{ kg/m}^3$$

$$F_p = \frac{1}{2} C_{DP} \rho A U^2$$

$$\Rightarrow [C_{DP} = 0.912]$$

③ Solⁿ:-

$$\text{Man}] W = 90 \text{ kgf} = 90 \times 9.81 = 882.9 \text{ N}$$

$$U = 20 \text{ m/s}$$

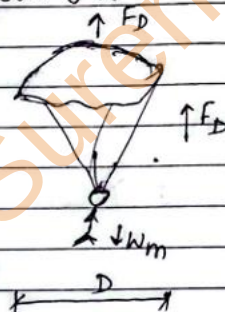
$$C_D = 0.5$$

$$\rho_{\text{air}} = 1.25 \text{ kg/m}^3$$

$$\text{Dia of parachute (D) = ?}$$

Wt. of parachute (neglected)

$$A = \frac{\pi \times D^2}{4} \text{ (Projected area)}$$



$$W_m = F_D = \frac{1}{2} C_D \rho A U^2$$

$$\text{or } \frac{\pi D^2}{4} = 7.063 \text{ m}^2$$

$$[D = 3 \text{ m/s}]$$

Drag force

Motion at Against air
(may be vertical)
or here

④ Solⁿ:-

$$A = \frac{\pi D^2}{4} = 4\pi \text{ m}^2$$

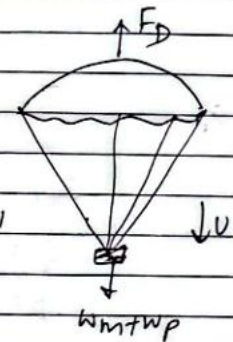
$$U = 25 \text{ m/s}$$

$$W_{\text{man}} = ?$$

$$W_{\text{parachute}} = W_p = 9.81 \text{ N}$$

$$C_D = 0.6$$

$$\rho_{\text{air}} = 1.25 \text{ kg/m}^3$$



$$F_D = W_m + W_p = \frac{1}{2} C_D \rho A U^2$$

$$\Rightarrow [W_m = 2.935 \text{ kN}]$$

⑤

$$W_{\text{rifle}} = 12.26 \text{ N}$$

$$A = 0.9 \text{ m}^2$$

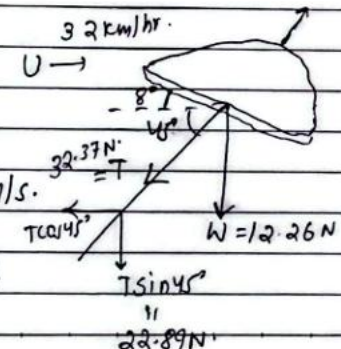
$$\text{Tension, } T = 32.37 \text{ N}$$

$$\theta = 45^\circ$$

$$U = 32 \text{ km/hr} = 8.89 \text{ m/s}$$

$$\rho_{\text{air}} = 1.203 \text{ kg/m}^3$$

$$C_D = ? \quad C_L = ?$$



$$F_D = T \cos 45^\circ = 22.89 = \frac{1}{2} C_D \rho A U^2$$

$$\Rightarrow [C_D = 0.535]$$

$$F_L = W + T \sin 45^\circ = 12.26 + 22.89 = \frac{1}{2} C_L \rho A U^2$$

$$\Rightarrow [C_L = 0.822]$$

⑥ Solⁿ:

$$W_{\text{kite}} = 1 \times 9.81 = 9.81 \text{ N}$$

$$T = 60 \text{ N}$$

$$U = 50 \text{ km/hr} = 13.89 \text{ m/s}$$

$$\rho_{\text{air}} = 1.2 \text{ kg/m}^3$$

$$C_L = 0.45$$

$$C_D = ?$$

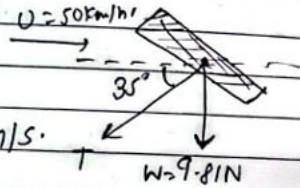
} Both coeff. based on full area of kite.

$$F_L = W + T \sin 35^\circ = 9.81 + 60 \sin 35^\circ = \frac{1}{2} C_L \rho A U^2$$

$$\Rightarrow [A = 0.849 \text{ m}^2]$$

$$\text{(opp. to flow)} \quad F_D = T \cos 35^\circ = \frac{1}{2} C_D \rho A U^2$$

$$\Rightarrow [C_D = 0.54]$$



⑦ Solⁿ:

Spherical ball,

$$D = 40 \text{ mm}$$

$$A = \frac{\pi D^2}{4} = 0.00126 \text{ m}^2$$

$$\text{Vol.} = \frac{4}{3} \pi R^3 = 33.51 \times 10^{-6} \text{ m}^3$$

$$\rho_{\text{ball}} = 8500 \text{ kg/m}^3$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$C_D = 0.45$$

Terminal velocity of ball in water, $U_w = ?$

For air, $C_D = 0.1$

$$\rho_{\text{air}} = 1.25 \text{ kg/m}^3$$

$$U_{\text{air}} = ? \quad \Delta U = ?$$

⑧ In water,

In eqⁿ,

$$F_D + F_{\text{buoyant}} = W$$

$$\text{or, } \frac{1}{2} C_D \rho_w A U_w^2 + \frac{4}{3} \pi R^3 \rho_w \times 9.81 = \frac{4}{3} \pi R^3 \rho_{\text{ball}} \times 9.81$$

$$\text{or, } \frac{1}{2} \times 0.45 \times 1000 \times 0.00126 \times U_w^2$$

$$+ 33.51 \times 10^{-6} \times 1000 \times 9.81$$

$$= 33.51 \times 10^{-6} \times 8500 \times 9.81$$

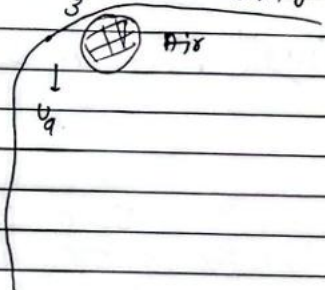
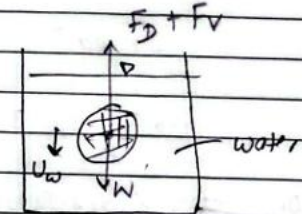
$$[U_w = 2.95 \text{ m/s}]$$

⑨ In air,

$$\frac{1}{2} \times 0.1 \times 1.25 \times 0.00126 \times U_{\text{air}}^2 + 33.51 \times 10^{-6} \times 1.25 \times 9.81$$

$$= 33.51 \times 10^{-6} \times 8500 \times 9.81$$

$$[U_{\text{air}} = 188.35 \text{ m/s}]$$



Increase in terminal velocity.
 $\Delta U = 185.4 \text{ m/s}$

⑧ Solⁿ:

Spherical ball,

$$D = 2 \text{ mm}$$

$$\gamma_{\text{ball}} = 117.5 \text{ kN/m}^3$$

$$\text{Vol.} = \frac{\pi D^3}{6} = 4.188 \times 10^{-9} \text{ m}^3$$

$$\text{Fluid } \mu = 15 \text{ poise} = 1.5 \text{ N s/m}^2$$

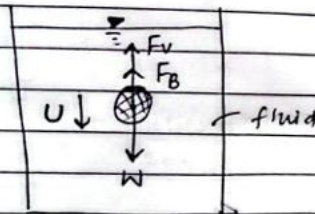
$$S = 0.95 \Rightarrow \rho = S \times 1000 = 950 \text{ kg/m}^3$$

Projected area (A) -

In eqⁿ.

viscous drag

$$W = F_B + (F_v)_D$$



$$\text{Or, } \text{Vol.} \times 117.5 \times 10^3 = \text{Vol.} \times 950 \times 9.81 + F_D$$

$$[F_D = 4.5 \times 10^{-4} \text{ N}]$$

For sphere

$$\text{Pressure Drag (} F_{DP} \text{)} = \frac{1}{3} \times F_D$$

$$= 1.5 \times 10^{-4} \text{ N} \quad \begin{matrix} \text{G.G.} \\ \text{Stoke} \end{matrix}$$

$$\text{Skin Drag (} F_{DP} \text{)} = \frac{2}{3} \times F_D$$

$$= 3 \times 10^{-4} \text{ N}$$

We know,

$$F_D = 3\pi \mu D U \quad (\text{Stoke's Formula})$$

$$\text{Or, } 4.5 \times 10^{-4} = 3\pi \times 1.5 \times 2 \times 10^{-3} \times U$$

$$\Rightarrow [U = 0.016 \text{ m/s}]$$

To use these formula,

$$\text{Check } Re = \frac{\rho U D}{\mu} = \frac{\rho U D}{\mu}$$

$$= \frac{0.016 \times 2 \times 10^{-3} \times 950}{1.5}$$

$$= 0.02 < 0.2$$

So, can use, (Stoke)

$$F_{DP} = \frac{1}{3} F_D$$

only for $Re < 0.2$

⑨ Solⁿ:

Sphere,

$$S = 1.2 \quad (\text{or } g)$$

$$D = 11 \text{ mm}$$

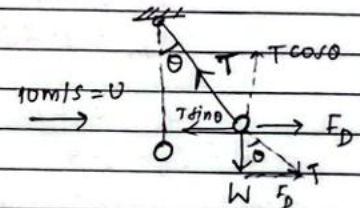
$$U = 10 \text{ m/s}$$

$$\rho_{\text{air}} = 1.203 \text{ kg/m}^3$$

$$\mu = 1.9 \times 10^{-5} \text{ Pa s}$$

$$A = \frac{\pi D^2}{4} = 95.03 \times 10^{-6} \text{ m}^2$$

$$\cos \theta = \frac{W}{T}$$



$$\tan \theta = \frac{F_D}{W} \quad \text{--- (1)}$$

$$W = \gamma \times \text{Vol.}$$

$$= 1.2 \times 1000 \times 696.91 \times 10^{-9} \text{ m}^3 \times 9.81$$

$$= 8.204 \times 10^{-3} \text{ N}$$

$$F_D = \frac{1}{2} C_D \rho_{\text{air}} A U^2 = \frac{1}{2} \times C_D \times 1.203 \times 95.03 \times 10^{-6} \times 10^2$$

$$F_D = 0.00572 C_D \text{ N}$$

Now,

$$Re = \frac{\rho U D}{\mu} = \frac{1.203 \times 10 \times 11 \times 10^{-3}}{1.9 \times 10^{-5}}$$

$$[Re = 6.96 \times 10^3]$$

For, $10^3 \leq Re \leq 10^5 \rightarrow C_D = 0.5$

So,

$$\theta = \cot^{-1} \left(\frac{8.204 \times 10^{-3}}{0.00572 \times 0.5} \right)$$

$$= 19.22^\circ$$

$$T \cos \theta = W$$

$$\Rightarrow [T = 0.0087 \text{ N}] \checkmark$$

(10) Soln.

Circular cylinder,



$$L = 1 \text{ m} \quad A = D \times L = 0.03 \text{ m}^2$$

$$D = 30 \text{ mm}$$

$$\rho_{\text{air}} = 1.236 \text{ kg/m}^3$$

$$U = 3.6 \text{ m/minute} = 0.06 \text{ m/s}$$

$$\text{Total } C_D = 1.4$$

$$\text{shear } (C_D)_f = 0.185$$

$$\text{Total Drag; } F_D = \frac{1}{2} C_D \rho A U^2$$

$$= \frac{1}{2} \times 1.4 \times 1.236 \times 0.03 \times 0.06^2$$

$$= 93.4416 \times 10^{-6} \text{ N}$$

$$\text{Shear Drag, } (F_D)_f = \frac{1}{2} (C_D)_f \rho A U^2$$

$$= 12.347 \times 10^{-6} \text{ N}$$

$$\text{Total Drag } (F_D) = (F_D)_f + (F_D)_p$$

$$\text{Pressure Drag; } F_{Dp} = 81.09 \times 10^{-6} \text{ N}$$

(11) Soln.

Cylinder,

$$L = 15 \text{ m}$$

$$D = 3 \text{ m}$$

$$U = 1.54 \text{ m/s}$$

$$\nu = 1.67 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\rho_{\text{water}} = 1025 \text{ kg/m}^3$$

$$A = L \times D = 15 \times 3 = 45 \text{ m}^2$$

$$Re = \frac{\rho U D}{\mu} = \frac{U D}{\nu} = \frac{1.54 \times 3}{1.67 \times 10^{-6}}$$

$$= 2.766 \times 10^6$$

(Take, $C_D = 0.7$)

So,

$$F_D = \frac{1}{2} C_D \rho A U^2$$

$$= 38.286 \text{ kN}$$

Friction Drag, $(F_D)_f = 0 \text{ N}$

∵ At high Reynold's number,

viscous effect is negligible

Thus, $(F_D)_f$ can be neglected

↓
due to viscosity

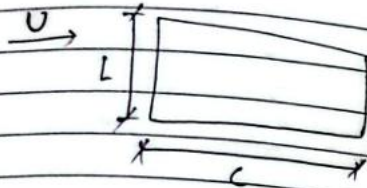
(12) Solⁿ:

Span length, $L = 10\text{ m}$
 Chord length, $C = 1.2\text{ m}$

$$A = L \times C = 12\text{ m}^2$$

$$U = 240\text{ km/hr}$$

$$= 66.67\text{ m/s}$$



Total aerodynamic force,
 resultant of lift & drag $\left\{ \begin{array}{l} F_L = 20\text{ kN} \\ F_D = 10 \end{array} \right.$

$$F_L = 10$$

$$F_D$$

$$C_D = ? ; C_L = ?$$

$$\rho_{\text{air}} = 1.2\text{ kg/m}^3$$

$$F_R^2 = F_D^2 + F_L^2$$

$$\text{or, } 20^2 = F_D^2 + (10)^2$$

$$\Rightarrow [F_D = 1.99\text{ kN}]$$

$$[F_L = 10 F_D = 19.90\text{ kN}]$$

$$[\text{Thrust, } P = F_D = 1.99\text{ kN}]$$

$$[\text{Wt., } W = F_L = 19.90\text{ kN}]$$

$$F_D = \frac{1}{2} C_D \rho A U^2 = 1.99$$

$$\frac{1}{2} C_D \rho A U^2 = 1.99$$

$$\frac{1}{2} C_D \rho A U^2 = 1.99$$

$$\frac{1}{2} C_D \rho A U^2 = 1.99$$

$$\Rightarrow [C_D = 0.0622]$$

$$[C_L = 0.622]$$

(13) Solⁿ:

Wing area, $A = 30\text{ m}^2$

Wt., $W = 28\text{ kN}$

$$U = 900\text{ km/hr} = 250\text{ m/s}$$

Power developed in engine = 8000 kW

70% power overcome drag resistance

$$P_D = 5600\text{ kW}$$

$$\rho_{\text{air}} = 1.2\text{ kg/m}^3 \quad C_D = ? \quad C_L = ?$$

We know,

$$P_D = F_D \times U$$

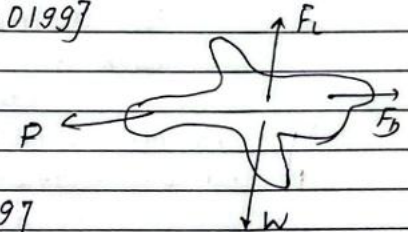
$$\text{or, } 5600 \times 10^3 = \frac{1}{2} \times C_D \times 1.2 \times 30 \times 250^2 \times 250$$

$$\Rightarrow [C_D = 0.0199]$$

$$F_L = W = 28 \times 10^3$$

$$\text{or, } \frac{1}{2} C_L \times 1.2 \times 30 \times 250^2 = 28 \times 10^3$$

$$\Rightarrow [C_L = 0.0249]$$



chapter - 14 :

①

Functional relationship

$$P = f(\rho, D, V, \omega, \mu, C) \quad P = K \rho \omega^3 D^5$$

corr. exponential relationship

$$P = K' \rho^a D^b V^c \omega^d \mu^e C^f$$

$K' \rightarrow$ dimensionless constant

$a, b, \dots, f \rightarrow$ exponents to be evaluated

Writing dimension of all variable,

$$[ML^2T^{-3}] = K' [ML^{-3}]^a [L]^b [LT^{-1}]^c [T^{-1}]^d [ML^{-1}T^{-1}]^e [LT^{-1}]^f$$

$$\mu = \frac{ML}{T} \\ = MLT^{-1}$$

$$\text{or } [M^1 L^2 T^{-3}] = K' [M^{a+e} L^{-3a+b+c-e+f} T^{-c-d-e-f}]$$

For dimensional homogeneity exponents of each dimension on both sides must be same/identical,

$$\text{For } M: a+e=1$$

$$\text{For } L: -3a+b+c-e+f=2$$

$$\text{For } T: -c-d-e-f=-3$$

Three eqs $\rightarrow$ 6 unknowns so,

3 unknown must be expressed in terms of other three:-

$$a = 1-e$$

$$c+f = 3-d-e$$

$$a = 1-e \quad \text{--- (i)}$$

$$b = 2+3(1-e)+e-(3-d-e)$$

$$b = 2+d-e \quad \text{--- (ii)}$$

$$f = 3-c-d-e \quad \text{--- (iii)}$$

in terms of d, e, c

Thus,

$$P = K' \rho^{1-e} D^{2+d-e} V^c \omega^d \mu^e C^{3-c-d-e}$$

$$P = K' \rho \omega^3 D^5 \left[\frac{\rho}{\rho} D^{d-e-3} \omega^{d-3} \mu^e V^c C^{3-c-d-e} \right]$$

$$P = K' \rho \omega^3 D^5 \left[\left(\frac{V}{C} \right)^c \left(\frac{\mu}{\rho D C} \right)^e \left(\frac{\omega D}{C} \right)^{d-3} \right]$$

$$P = \rho \omega^3 D^5 f_1 \left[\left(\frac{V}{C} \right), \frac{\mu}{\rho D C}, \frac{\omega D}{C} \right]$$

$$\frac{V}{C}, \frac{\mu}{\rho D C}, \frac{\omega D}{C} \rightarrow \text{dimensionless \& number}$$

$$[P = K \rho \omega^3 D^5]$$

$K \rightarrow$ dimensionless constant.

②

Solⁿ:

$$R = \rho D^3 V^2 f(\mu/\rho V D)$$

$$R = \mu V D \left(\frac{\rho V D}{\mu} \right) f(\mu/\rho V D)$$

$$R = \mu V D \left(\frac{\mu}{\rho V D} \right)^{-1} f(\mu/\rho V D)$$

$$R = \mu V D f(\mu/\rho V D)$$

$$[R = K \mu V D]$$

$K \rightarrow$ dimensionless constant.

(3)

$$F = f(D, V, \rho, \mu, t)$$

$$\text{or, } f_1(F, D, V, \rho, \mu, t) = 0$$

$$n = 6$$

$$m = 3$$

$$\Rightarrow n - m = 3 \text{ } \pi \text{ terms}$$

$$f_2(\pi_1, \pi_2, \pi_3) = C \quad \left. \begin{array}{l} C \rightarrow \text{dimensionless} \\ \text{const} \end{array} \right\}$$

$$\pi_1 = \Phi_1(D, V, \rho, F) \quad \pi \rightarrow \text{Dimensionless quantity.}$$

$$\pi_2 = \Phi_2(D, V, \rho, \mu) \quad \rho V D \neq \underline{MLT^{-2}}$$

$$\pi_3 = \Phi_3(D, V, \rho, t)$$

$$\text{For } \pi_1, \quad \pi_1 = D^{a_1} V^{b_1} \rho^{c_1} F$$

$$M^0 L^0 T^0 = L^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} (MLT^{-2})$$

$$M^0 L^0 T^0 = M^{c_1+1} L^{a_1+b_1-3c_1+1} T^{-b_1-2} \quad \text{--- (i)}$$

$$\text{For } \pi_2, \quad \pi_2 = D^{a_2} V^{b_2} \rho^{c_2} \mu$$

$$M^0 L^0 T^0 = L^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} ML^{-1} T^{-1}$$

$$M^0 L^0 T^0 = M^{c_2+1} L^{a_2+b_2-3c_2-1} T^{-b_2-1} \quad \text{--- (ii)}$$

For π_3

$$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} t$$

$$\text{or, } M^0 L^0 T^0 = L^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} T$$

$$\text{or, } M^0 L^0 T^0 = M^{c_3} L^{a_3+b_3-3c_3} T^{-b_3+1} \quad \text{--- (iii)}$$

Comparing dimensions on R.H.S. & L.H.S. of (i), (ii), (iii).

$$a_1 = -2, b_1 = -2, c_1 = -1$$

$$a_2 = -1, b_2 = -1, c_2 = -1$$

$$a_3 = -1, b_3 = 1, c_3 = 0$$

Thus,

$$f(\pi_1, \pi_2, \pi_3) = C$$

$$\text{or, } f\left(\frac{F}{D^2 V^2 S}, \frac{\mu}{SVD}, \frac{Vt}{D}\right) = C$$

$$\text{or, } \frac{F}{D^2 V^2 S} = f_3\left(\frac{\mu}{SVD}, \frac{Vt}{D}\right)$$

$$\text{or, } F = S V^2 D^2 \phi\left(\frac{\mu}{SVD}, \frac{Vt}{D}\right)$$

(4) Solⁿ:

$$Q = f_1(\alpha, H, g, S, \mu, \sigma)$$

$$\Rightarrow f_2(Q, \alpha, H, g, S, \mu, \sigma) = C_1 \quad \text{--- (i)}$$

$n = 6$ (dimensional var)

$m = 3$

$n - m = 3$ π terms.

α is π -term itself being dimensionless quantity.

Other π -terms

$$\pi_1 = f_1(H, S, g, Q)$$

$$\pi_1 = H^{a_1} S^{b_1} g^{c_1} Q \quad \text{--- (i)}$$

$$\pi_2 = f_2(H, S, g, \mu)$$

$$\pi_2 = H^{a_2} S^{b_2} g^{c_2} \mu \quad \text{--- (ii)}$$

$$\pi_3 = f_3(H, S, g, \sigma)$$

$$\pi_3 = H^{a_3} S^{b_3} g^{c_3} \sigma \quad \text{--- (iii)}$$

$$f_3(\pi_1, \pi_2, \pi_3, \alpha) = C_2 \quad \text{--- (iv)}$$

$$\sigma = \frac{F}{S}$$

In dimension form

$$\begin{aligned} \pi_1: M^0 L^0 T^0 &= L^{a_1} [ML^{-3}]^{b_1} [LT^{-2}]^{c_1} [L^3 T^{-1}] \\ M^0 L^0 T^0 &= M^{b_1} L^{a_1 - 3b_1 + c_1 + 3} T^{-2c_1 - 1} \end{aligned}$$

$$\begin{aligned} \pi_2: M^0 L^0 T^0 &= L^{a_2} [ML^{-3}]^{b_2} [LT^{-2}]^{c_2} [ML^{-1} T^{-1}] \\ M^0 L^0 T^0 &= M^{b_2 + 1} L^{a_2 - 3b_2 + c_2 - 1} T^{-2c_2 - 1} \end{aligned}$$

$$\begin{aligned} \pi_3: M^0 L^0 T^0 &= L^{a_3} [ML^{-3}]^{b_3} [LT^{-2}]^{c_3} [MT^{-2}] \\ M^0 L^0 T^0 &= M^{b_3 + 1} L^{a_3 - 3b_3 + c_3} T^{-2c_3 - 2} \end{aligned}$$

Comparing exponents in above term,

$$a_1 = -5/2, \quad b_1 = 0, \quad c_1 = -1/2$$

$$a_2 = -3/2, \quad b_2 = -1, \quad c_2 = -1/2$$

$$a_3 = -2, \quad b_3 = -1, \quad c_3 = -1$$

from (iv).

$$f_3\left(\frac{Q}{H^{5/2} g^{1/2}}, \frac{\mu}{H^{3/2} S g^{1/2}}, \frac{\sigma}{H^2 S g}, \alpha\right) = C_2$$

$$\text{or, } \frac{Q}{H^{5/2} g^{1/2}} = f_4\left(\frac{\mu}{H^{3/2} S g^{1/2}}, \frac{\sigma}{H^2 S g}, \alpha\right)$$

$$\text{or, } Q = H^{5/2} \sqrt{g} f_4\left(\frac{\mu}{H^{3/2} S g^{1/2}}, \frac{\sigma}{H^2 S g}, \alpha\right)$$

$$\text{or, } [Q = C H^{5/2}] \text{ as } g \text{ is constant value.}$$

⑤

Solⁿ:-

$$\frac{L_m}{L_p} = \frac{1}{10}$$

$$V_p = 8 \text{ m/s}$$

$$V_m = ?$$

$$V_p \cdot L_p = V_m \cdot L_m$$

$$V_m = 1 \times 10^{-6} \text{ m/s}$$

$$\rho_p = 1027 \text{ kg/m}^3$$

$$\rho_m = 1000 \text{ kg/m}^3$$

From Reynold's Model law

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p} \quad \text{--- (1)}$$

$$\text{or } \frac{V_m L_m}{\mu_m} = \frac{V_p L_p}{\mu_p}$$

$$\text{or } V_m = \left(\frac{L_p}{L_m} \right) \left(\frac{\mu_m}{\mu_p} \right) \times V_p$$

$$= \left(\frac{10}{1} \right) \left(\frac{1 \times 10^{-6}}{1.121 \times 10^{-6}} \right) \times 8$$

$$= 71.37 \text{ m/s}$$

From (1)

Drag $\rightarrow$ viscous drag

$$\frac{\mu_m}{\mu_p} = \left(\frac{\rho_m}{\rho_p} \right) \left(\frac{V_m}{V_p} \right) \left(\frac{L_m}{L_p} \right)$$

$$F_D = \frac{1}{2} C_D \rho A V^2$$

=

Tested in tunnel.
Actual $\rightarrow$ sea.

⑥

Solⁿ:-

$$D_p = 1 \text{ m}$$

$$S_p = 0.8$$

$$\mu_p = 0.02 \text{ poise}$$

$$Q_p = 2000 \text{ litres/sec} = 2 \text{ m}^3/\text{sec}$$

$$D_m = 0.1 \text{ m}$$

$$S_m = 1$$

$$V_m = 1 \times 10^{-4} \text{ m/s}$$

$$\mu_m = 1.0016 \times 10^{-3} \text{ Pa/s} = 1.0016 \times 10$$

$$\mu_m = 11.4 \times 10^{-3} \text{ poise}$$

$$(H_L)_m = 4.8 \text{ m of water} \quad L_m = 30 \text{ m}$$

$$(H_L)_p = ?$$

Reynold's eqⁿ model.

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\rho = S \times \rho_w$$

$$\frac{V_m}{V_p} = \left(\frac{D_p}{D_m} \right) \left(\frac{\mu_m}{\mu_p} \right) \left(\frac{\rho_p}{\rho_m} \right)$$

$$\frac{Q_m}{Q_p} \times \frac{A_p}{A_m} = \frac{1}{0.1} \times \frac{11.4 \times 10^{-3}}{0.02} \times \frac{0.8}{1}$$

$$\approx 4.56$$

$$Q_m = 4.56 \times \frac{0.1^2}{1^2} \times 2$$

$$[Q_m = 0.0912 \text{ m}^3/\text{s}]$$

$$[\text{For } \mu_m = 10 \times 10^{-3} \text{ poise, } Q_m = 0.08 \text{ m}^3/\text{s}]$$